Trade Policy and Global Sourcing: A Rationale for Tariff Escalation*

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Abstract

We study trade policy in an environment with intermediate and final good trade. Real-world import tariffs tend to be higher for final goods than for inputs, a phenomenon commonly referred to as tariff escalation. Yet, neoclassical trade theory – and modern Ricardian trade models, in particular – cannot easily rationalize this fact. We show that tariff escalation can be rationalized on efficiency grounds in the presence of scale economies. A unilateral tariff in either sector increases a country's relative wage and boosts the size and productivity of each sector, both of which raise welfare. While these forces are reinforced up the chain for final-good tariffs, input tariffs raise final-good producers' costs, mitigating their potential benefits. A quantitative evaluation of the US-China trade war demonstrates that any welfare gains from the increase in US tariffs are overwhelming driven by final-good tariffs.

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1 Introduction

Import tariffs tend to be lower on intermediate inputs than on final goods. This pattern has been documented across many countries in numerous studies that now span almost five decades (Travis, 1964; Balassa, 1965; Bown and Crowley, 2016; Shapiro, 2020), and is commonly referred to as 'tariff escalation,' a term that captures the fact that tariffs 'escalate' down the production chain. Building on data assembled by Shapiro (2020), Figure 1 plots average country-pair final-good versus input tariffs in 2007 and illustrates the prevalence of tariff escalation across trading partners.[1] For the vast majority of country pairs, the (unweighted) average tariff on final goods is higher than the average tariffs on inputs.

Not only are relatively low input tariffs a salient characteristic of trade policy, empirical work suggests that they improve firm and worker outcomes in downstream sectors. Early work documents significant productivity gains from lower input tariffs (Amiti and Konings, 2007; Goldberg et al., 2010; Topalova and Khandelwal, 2011), while new evidence shows that recent US input tariff hikes harmed US manufacturing employment (Flaaen and Pierce, 2019) and exports (Handley et al., 2020).[2]

Despite the ubiquity of tariff escalation and mounting evidence on the benefits of relatively low input tariffs, existing trade theory does not suggest that it is a welfare-maximizing policy. Early neoclassical models with homogeneous goods analyze input versus final-good tariffs explicitly, but do not show that optimal input tariffs should be relatively lower. Modern Ricardian trade models stress that first-best trade policy entails uniform tariffs across sectors, and that tariff *de*-escalation may be optimal in second-best settings without export taxes. Our reading is that the leading theoretical explanation for tariff escalation relies on political counter-lobbying (Cadot et al., 2004; Gawande et al., 2012), in which all firms lobby for protection of their output, but final-good producers counter-lobby against tariffs on their imported inputs.[3] If this counter-lobbying accounts for relatively lower input tariffs, then tariff escalation is not the result of social welfare-maximizing policy and instead reflects political capture by domestic producers.

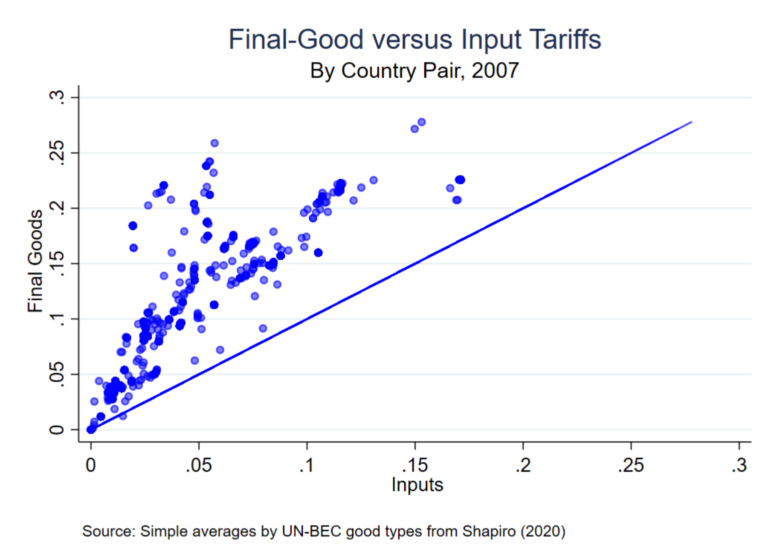
In this paper, we provide a social-welfare maximizing rationale for tariff escalation. Since neoclassical models featuring constant returns to scale do not predict that lower input tariffs are optimal, we use a general equilibrium framework with an upstream and downstream sector that

[1] Shapiro (2020) reports a smooth, negative correlation between industry-level tariffs and upstreamness, as measured by Antràs et al. (2012).

[2] Bown et al. (2020) study the impact of US antidumping duties against Chinese imports from 1988 to 2016 (and during the recent US-China trade war), and find negative effects on employment in downstream industries, with no counterbalancing positive effects in protected industries. Barattieri and Cacciatore (2020) present similar results for the period 1994-2015. Flaaen et al. (2020) document the relocation of production in response to tariffs on washing machines. Breinlich et al. (2021) show that the normalization of US trade relations with China (PNTR) led to higher US export growth due to lower costs on imported inputs from China. Cox (2021) finds that upstream steel tariffs enacted by President Bush in 2002 and 2003 had highly persistent negative impacts on the competitiveness of U.S. downstream industry exports.

[3] While consumers prefer lower tariffs on final-goods, the counter-lobbying explanation assumes that collective action disincentives preclude them from lobbying. This rationale seems at odds with the fact that consumer goods are increasingly imported by large wholesale and retail firms that dominate their sectors (Ganapati, 2018; Smith and Díaz, 2020).

Figure 1: Average Final-Good versus Input Tariffs



each produce differentiated inputs and final goods, respectively, under increasing returns to scale. Although we start by modeling these sectors as monopolistically competitive with scale economies being internal to firms, we show that there is an isomorphic model with perfect competition and external economies of scale that generates identical tariff motives. This isomorphism highlights our key finding. Tariff escalation is optimal when the downstream sector produces under increasing returns to scale, and the degree of escalation is increasing in the extent of those returns to scale.

We first solve for the decentralized equilibrium of the closed-economy model and show that it is inefficient whenever both sectors use labor to produce. The social planner can restore efficiency by subsidizing upstream production, which increases the share of labor upstream and thus the size of both sectors. While it might seem surprising that final-good production increases from shifting labor upstream, this occurs because the downstream sector also uses inputs to produce, and the expanded availability of inputs raises its labor efficiency. The optimal upstream subsidy is related to the elasticity of substitution between inputs (in the monopolistic competition version of the model), or similarly, to the external economies of scale (EOS) parameter – or *scale elasticity* – for upstream production in the isomorphic competitive model with external economies of scale. In both versions of the model, upstream firms do not internalize their impact on downstream efficiency, and the extent to which input availability affects final-good productivity is increasing in the inverse of the elasticity of substitution between inputs, or the EOS parameter.

To analyze tariff escalation, we first extend the vertical model to a small, open economy setting with a Home and Foreign country, and trade in both inputs and final goods. Home has market power over its exported varieties, but takes import prices (net of import tariffs) as given.[4] To analyze optimal policies, we follow the primal approach in Lucas and Stokey (1983) and Costinot et al. (2015). Specifically, we characterize the optimal allocation from a social planner's perspective,

[4]The assumption on Foreign price indices being fixed allows us to derive analytic solutions for optimal trade policy.

compare this solution to the decentralized market equilibrium with trade taxes, and finally show how trade taxes can be used to achieve the optimal allocations.

To build intuition, we first show that when the downstream sector does not use labor (i.e., it only assembles inputs), there is no domestic inefficiency and thus no need for production subsidies to achieve the first-best allocation. Instead, the first-best can be achieved using only a downstream tariff and an upstream export tax. Although Lerner symmetry implies that tariff levels can be scaled up or down arbitrarily, tariff escalation (the ratio of final-good to input tariffs) is uniquely pinned down, greater than one, and depends only on the degree of increasing returns to scale in downstream production.

The result that tariff escalation depends only on the downstream scale elasticity highlights a key distinction between the welfare effects of input versus final-good tariffs. Precisely in line with the empirical work described above, lower input tariffs increase domestic final-good producers' efficiency. Crucially, however, this efficiency gain is due not only to lower input costs, but also to the presence of increasing returns in downstream production. In fact, we show that tariff escalation persists as an optimal policy even when inputs are produced under constant returns to scale, whereas optimal input and final-good tariffs are uniform if final-good production does not feature increasing returns to scale.

Because export taxes are illegal in many countries and production subsidies are constrained by World Trade Organization (WTO) rules, we also analyze the scope for tariff escalation under a second-best environment in which the planner is limited to using only import tariffs. In this setting, tariff escalation is still optimal, and in fact even larger than under the first-best policies described above. Although the planner now uses a positive input tariff to exploit Home's market power abroad, the negative impact of the input tariff on final-good production must be offset using an even higher final-good tariff. Intuitively, the final-good tariff increases the size of Home's downstream sector, and the downstream tariff must now do this to a larger degree since the positive input tariff raises Home's input costs and provides an incentive for final-good producers to relocate to Foreign.[5]

Having developed intuition on tariff escalation in a simple setting in which all labor is employed upstream, we next analyze optimal policy when labor is used in both sectors. As in the closed-economy, the upstream sector is too small and an input subsidy is required to restore efficiency. In fact, if the government sets an upstream production subsidy at the same level as the optimal subsidy in the closed economy, the first-best can be achieved with only two additional trade taxes: a downstream import tariff and an upstream export tax, with the level of these instruments being identical to that obtained in the case in which the downstream sector does not use labor. As before, the final-good tariff, and thus tariff escalation, both depend on the extent of increasing returns in the downstream sector. Whenever productivity downstream increases in the size of the sector, the planner has an incentive to shift domestic consumption towards domestic firms. While the planner could use a downstream production subsidy instead of a tariff to do this, its use requires an

[5]This distinction highlights another crucial distinction between input versus final-good tariffs. While final-good producers can (and do) relocate to Foreign in response to input tariffs, Home consumers cannot move.

additional export tax downstream. Since our goal is to show whether and when tariff escalation emerges as a feature of welfare-maximizing trade policy, we focus on the tariff implementation.[6]

We also analyze the second-best optimal import tariffs when labor is used in both sectors and the government cannot use subsidies or export taxes. Unlike in the simpler setting in which all labor was employed upstream, in this case the government has a welfare-motive for an input tariff since it increases upstream entry, thereby mimicking the missing upstream subsidy. This works against the forces that led to a magnified second-best tariff escalation for the case in which the downstream sector does not use labor. While these opposing forces have precluded us from obtaining an analytic characterization of tariff escalation under the second-best in this setting, extensive numerical simulations suggest that the subsidy-role of the upstream tariff is dominated by the importance of downstream returns to scale, such that tariff escalation is always greater than one.

We close our theoretical analysis by relaxing the small open economy assumption on Foreign prices so that we can decompose the first-order welfare effects of a *small* input or final-good tariff into a relative wage effect, a relocation effect, and an export tax proxy effect. As in standard Ricardian models, a small tariff in either sector increases the relative demand for Home goods in that sector, which in turn raises Home's relative wage. Our framework further allows for tariffs to affect welfare via changes in the mass of firms in each sector. An increase in a sector's tariff increases entry in that sector, which raises its efficiency due to increasing returns, and thus leads to a lower price index for the sector, and higher welfare. These gross relocation effects are similar to those in Venables (1987) and Ossa (2011) for final-goods, but by modeling them in general equilibrium, we also show that they are comparable in size to the relative wage effects in traditional trade models. A key distinction in our setting is that a tariff in one sector reallocates domestic labor from the other sector, which generates an offsetting negative relocation effect.

Quantification exercises indicate that the net relocation effects of a final-good tariff are positive, whereas they are negative for input tariffs. An input tariff increases the size and thus efficiency of the upstream sector, but does so at the expense of pulling labor from the downstream sector, which raises domestic final-good prices and lowers welfare. By contrast, the downstream tariff increases domestic final-good production, which raises not only wages but also demand for domestic inputs. Finally, the decomposition shows that an input tariff acts as an indirect export tax on final goods by raising domestic input prices, which leads to an 'export tax proxy' motive for input tariffs described in Beshkar and Lashkaripour (2020). In our quantification, we find that this effect is relatively small compared to the terms-of-trade and relocation effects.

Having built intuition via our theoretical analysis, we calibrate the monopolistic competition model to fit World Input Output data on the United States and the Rest of the World (RoW) in 2014. We first use the same elasticity of substitution for final goods and inputs to ensure that an asymmetry in those parameters does not drive our results, and then perform numerous sensitivity analyses with different values for these parameters. We use the calibrated model to quantify optimal

[6] The planner could also restore efficiency using an upstream consumption subsidy. If the planner uses a non-discriminatory consumption subsidy on both domestic and foreign input purchases, then he must offset the subsidy of Home's foreign input purchases using an offsetting input tariff.

trade policy, and to decompose the channels through which tariffs affect welfare. When the Home government's policy tools are limited to import tariffs, we find that the optimal tariffs are 30.6 percent on final goods, versus only 17.0 percent on inputs. This tariff escalation is evident across a wide range of elasticity values, labor shares, and differences in the size of Home versus the RoW. We also use the calibrated model to characterize optimal trade policy when Home has access to additional policy instruments, such as export taxes or domestic upstream production subsidies, that yield the first-best allocation. Tariff escalation under the first-best set of policies in this large, open economy setting is quite close to our analytic solution for the small, open economy. In the second-best, tariff escalation is decreasing in the extent to which the downstream sector uses labor in production, consistent with premise that an input tariff mimics the missing upstream subsidy.

Finally, we use the model to study how welfare responds to the US tariff increases imposed during the Trump presidency. Approximately 60 percent of the Trump tariffs through 2018 were on inputs, affecting nearly 20 percent of all US imports of intermediate inputs (Bown and Zhang, 2019). Our quantitative results indicate that, absent any foreign retaliation, the increase in US tariffs would have raised US welfare by 0.12 percent, but that this positive effect is overwhelmingly driven by higher final-good tariffs. Taking into account the change in foreign retaliatory tariffs, the increase in US welfare shrinks to 0.02 percent. By comparison, the welfare effects net of retaliation would have been negative if input tariffs alone had been used.

Our paper contributes to the literature in several ways. First, we provide an efficiency rationale for tariff escalation in an environment with a benevolent social planner.[7] Early work using neoclassical models featuring homogeneous final goods and inputs (Ruffin, 1969; Casas, 1973; Das, 1983) explicitly modeled both types of tariffs, but did not predict that optimal input tariffs should necessarily be lower than final-good tariffs, despite their productivity-enhancing effects. Recent work on optimal trade policy in multi-sector competitive Ricardian models deliver predictions inconsistent with this pattern (Costinot et al., 2015; Beshkar and Lashkaripour, 2020).[8] Also working with a competitive model, Blanchard et al. (2021) demonstrate that the terms-of-trade motive for final-good tariffs persists, but is dampened, when a country's final-good imports contain its domestic value added. A key difference between these recent studies and ours is that we model input versus final-good production separately, and that we allow for trade and tariffs on both types of goods. Unlike in models of roundabout production, this vertical structure allows us to characterize how optimal tariffs differ for inputs versus final goods, and to decompose the distinct channels through which they affect welfare.[9]

[7]Our result on tariff escalation differs from the celebrated production efficiency result in Diamond and Mirrlees (1971), since that applies to a closed-economy setting in which a planer seeks to raise a given amount of government revenue at the minimum efficiency cost. Tariff escalation is also distinct from the phenomenon of cascading trade protection (see Erbahar and Zi, 2017), which is concerned with the effects of upstream tariffs on the *demand* for downstream tariffs, regardless of efficiency considerations.

[8]Since the terms-of-trade motive is decreasing in a sector's export supply elasticity, the tariff escalation present in real-world trade policy might appear to be consistent with existing neoclassical theory if export supply elasticities were to be higher for intermediate inputs than for final goods. However, the data used in Shapiro (2020) indicate a weak positive correlation of 0.049 between the measure of upstreamness in Antràs et al. (2012) and the inverse export supply elasticities in Soderbery (2015).

[9]Further away from our focus, the effects of tariffs on intermediate inputs in a world with *relational* GVCs have

Our departure from perfect competition and constant returns to scale also highlights the potential for final-good and input tariffs to affect welfare by changing the mass of firms in both sectors. These production relocation effects are studied in Venables (1987) and Ossa (2011), who show that with imperfect competition and scale economies, an increase in a final-good import tariff attracts firms to a country, which in turn lowers prices and thus raises welfare. We extend the analysis by adding input trade and moving from a partial- to a general-equilibrium framework. Prior work finds that in general equilibrium with roundabout production, agglomeration forces may lead countries to specialize in manufacturing production (Krugman and Venables, 1995; Puga and Venables, 1999). Amiti (2004) introduces a two-sector model with agglomeration forces and uses numerical simulations to argue that tariff de-escalation is optimal. Our contribution is to model input versus final-good sectors separately so that we can study tariff escalation explicitly, and to show both analytically and numerically when and why tariff escalation is optimal.

Finally, we add to a growing body of work that studies optimal trade policy in the presence of market power and domestic distortions. Early work shows that market power provides an incentive for final-good tariffs, even when countries are too small to affect world prices (Gros, 1987; Demidova and Rodiguez-Clare, 2009).[10] Caliendo et al. (2021) demonstrate that when domestic subsidies are unavailable, the optimal tariff is lower when imports are used in production in a roundabout setting, because the lower tariff mitigates a double marginalization inefficiency that arises from markups in the tradable sector. Our results differ because we model trade in both inputs and final-goods and show that tariff escalation *decreases* when an upstream subsidy is not available. We also exploit an isomorphism between the model with market power and one with external economies of scale, similar to Kucheryavyy et al. (2017), to show analytically that the first-best allocation features tariff escalation. This may seem at odds with Lashkaripour and Lugovskyy (2021), who characterize optimal tariffs as independent of the economy's input-output structure when optimal production subsidies are available. While we also find that a downstream production subsidy can be used instead of a downstream tariff, its use then requires a downstream export tax to offset the portion of the subsidy that accrues to foreign consumers. In sum, we show that optimal tariffs on inputs are lower than those on final-goods when using the minimum number of domestic instruments necessary to achieve the first-best. Extensive robustness tests indicate that this is true for a wide range of parameter values.[11]

The rest of the paper is structured as follows. In section 2, we use a closed-economy version of our Krugman-style model expanded to include an intermediate-input sector to analyze domestic distortions and optimal policy. In section 3, we develop the open-economy version of the model, as well as the isomorphism to a competive economy with external economies of scale. Optimal policy

also been studied in frameworks with incomplete contracting by Ornelas and Turner (2008), Antràs and Staiger (2012), Ornelas and Turner (2012), and Grossman and Helpman (2020).

[10] See Campolmi et al. (2014, 2018) for other recent work on optimal trade policy in the presence of domestic distortions. Other papers study the desirability of tariff escalation under various market structures (e.g., Spencer and Jones, 1991, 1992; McCorriston and Sheldon, 2011; Hwang et al., 2017).

[11] In this sense, our paper contributes to an active literature studying the quantitative implications of trade policy (Eaton and Kortum, 2002; Alvarez and Lucas Jr, 2007; Costinot and Rodríguez-Clare, 2014; Ossa, 2014).

for a small open economy is developed in sections 4 and 5. Section 6 studies the impact of small input and final-good tariffs for a large open economy. We finally develop our quantitative results in section 7 and conclude in section 8.

2 A Krugman Economy with an Input Sector

In this section, we introduce a closed-economy model with an upstream (input) sector and a downstream (final-good) sector, both featuring increasing returns to scale, product differentiation, and monopolistic competition. Our framework boils down to a simple extension of the closed-economy version of the classical model in Krugman (1980), expanded to include an intermediate-good sector. We begin with this relatively simple framework because we can derive analytical solutions to both the market equilibrium and the social planner's problem, which provide intuition for how imperfect competition and scale economies in vertical sectors lead to welfare-reducing distortions.

2.1 Environment

Consider an economy in which the representative consumer values the consumption of differentiated varieties of manufacturing goods according to the utility function

$$U = \left(\int_0^{M^d} q^d\left(\omega\right)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}, \qquad \sigma > 1 \tag{1}$$

where M^d is the endogenous measure of final-good varieties produced in the economy, $q^d\left(\omega\right)$ is the quantity consumed of variety ω, and σ is the elasticity of substitution across varieties. Individuals supply one unit of labor inelastically, with L denoting the total labor force. There are no other factors of production, so labor should be interpreted as representing "equipped" labor.

Labor is used for the production of intermediate inputs (the upstream sector) and (possibly) in producing final goods (the downstream sector). More specifically, we represent technologies in the upstream and downstream sectors with

$$f^u + x^u(\varpi) = A^u \ell^u(\varpi), \qquad \varpi \in [0, M^u], \tag{2}$$

and

$$f^d + x^d(\omega) = A^d \ell^d(\omega)^{\alpha} Q^u(\omega)^{1-\alpha}, \qquad \omega \in [0, M^d], \quad \alpha \in [0,1], \tag{3}$$

respectively. In these expressions, f^s denotes the fixed output requirements for entry in sector $s \in \{D, U\}$, $x^s(\omega)$ is the output produced for sale by variety ω in sector $s \in \{D, U\}$, A^s is a sector-specific technology parameter, and $Q^u(\omega)$ is a composite of all intermediate goods, which is in turn given by

$$Q^u(\omega) = \left(\int_0^{M^u} q^u(\varpi)^{\frac{\theta-1}{\theta}} d\varpi \right)^{\frac{\theta}{\theta-1}}, \qquad \theta > 1, \tag{4}$$

where $q^u(\varpi)$ is the quantity consumed of input variety ϖ. In words, the upstream sector uses only labor, and technology features increasing returns to scale due to the presence of a fixed overhead cost. The downstream sector combines labor with a continuum of intermediate inputs of measure M^u, where M^u is endogenous, and technology again exhibits increasing returns stemming from a fixed overhead cost. Notice that $\theta > 1$ governs the degree of substitutability across inputs, while $\alpha \in [0, 1]$ corresponds to the downstream labor (or value-added) intensity in production.

There is an endogenous measure, M^d, of manufacturing firms in the downstream sector, each producing a single final-good variety. Analogously, there is an endogenous measure, M^u, of manufacturing firms in the upstream sector, each producing a single intermediate-input variety. All entrants have access to the same technologies in (2), (3) and (4). Market structure in both sectors is characterized by monopolistic competition and free entry.

2.2 Equilibrium and Efficiency

Given the CES assumptions built into our framework and the lack of strategic interactions, firms in both sectors charge a constant markup over their marginal cost, which combined with free entry, pins down firm size according to (see Online Appendix A for details):

$$x^u = (\theta - 1) f^u, \qquad x^d = (\sigma - 1) f^d. \tag{5}$$

Naturally, in equilibrium we must have $x^d = q^d$ and $x^u = M^d q^u$. Invoking households' demand for downstream goods and labor-market clearing (see Online Appendix A), we can determine the measure of upstream and downstream firms in the economy:

$$M^u = \frac{(1-\alpha) A^u L}{f^u \theta}; \tag{6}$$

$$M^d = \frac{\alpha^\alpha A^d}{f^d \sigma} \left((\theta - 1) f^u \right)^{1-\alpha} \left(\frac{(1-\alpha) A^u}{f^u \theta} \right)^{\frac{(1-\alpha)\theta}{\theta - 1}} (L)^{\frac{\theta - \alpha}{\theta - 1}}. \tag{7}$$

Finally, welfare of the representative consumer is simply given by $U = \left(M^d \right)^{\frac{\sigma}{\sigma - 1}} q^d$, where M^d is given in (7) and $q^d = x^d$ in (5). When $\alpha \to 1$, we obtain

$$U = \left(\frac{A^d}{f^d \sigma} L \right)^{\frac{\sigma}{\sigma - 1}} (\sigma - 1) f^d,$$

which is the standard formula in Krugman (1980).[12] Welfare is increasing in market size with an elasticity equal to $\sigma/(\sigma - 1) > 1$, reflecting the variety gains associated with living in an economy that provides a larger number of final-good varieties.

Relative to this "Krugman" benchmark, in the presence of an active upstream sector (i.e., $\alpha < 1$),

[12] A small and immaterial point of departure from Krugman (1980) is the fact that we have modeled the productivity terms A^d and A^u as shaping both the marginal and fixed costs of production. As a result, firm size is independent of these productivity parameters, but these parameters affect welfare directly.

our model continues to feature scale effects, and in fact the elasticity of welfare to market size is larger than in the model with only a final-good sector. To see this, we can write welfare as

$$U = \left(\frac{(\sigma-1)A^d/\sigma}{((\sigma-1)f^d)^{\frac{1}{\sigma}}} \left(\frac{(\theta-1)\,A^u/\theta}{((\sigma-1)f^u)^{1/\theta}} \right)^{\frac{(1-\alpha)\theta}{\theta-1}} (L)^{\frac{\theta-\alpha}{\theta-1}} \right)^{\frac{\sigma}{\sigma-1}} \xi_\alpha, \tag{8}$$

where ξ_α is a function of only α and θ. Note that $\frac{\theta-\alpha}{\theta-1} \geq 1$, and thus the elasticity of welfare with respect to L is larger when $\alpha < 1$.

To gain a better understanding of the role of imperfect competition and increasing returns to scale in shaping welfare in our closed economy, in Online Appendix A.2 we characterize the social optimum in our model, and explore conditions under which the above market equilibrium is efficient. There, we prove the following result:

Proposition 1. In the decentralized equilibrium, firm-level output is at its socially optimal level in both sectors, but the market equilibrium features too little entry into both the downstream and upstream sectors unless $\alpha = 1$ (so the upstream sector is shut down) or $\alpha = 0$ (so the downstream sector does not use labor directly in production).

Why is the market equilibrium typically inefficient? It might seem intuitive that this inefficiency is associated with upstream markups leading to a double-marginalization inefficiency. However, the market allocation of labor to the upstream sector is given by $(1-\alpha)L$ and is in fact independent of the degree of input substitutability (θ) and thus of the level of upstream markups. In other words, lower input substitutability – and thus higher markups – do not depress the market allocation of labor to the upstream sector; instead, they increase the social-welfare maximizing allocation of labor to that sector (see Online Appendix A.2). This fact does not necessarily rule out the relevance of a double marginalization inefficiency, but it does suggest that the market inefficiency may alternatively be interpreted as reflecting that, in the market equilibrium, upstream firms do not internalize the fact that their entry generates positive spillovers for firms in the downstream sector, with the size of this spillover decreasing in the degree of input substitutability θ.

To reinforce this interpretation, in Online Appendix A.4, we show that the equilibrium of our vertical Krugman economy is isomorphic to that of a competitive vertical economy with external economies of scale under specific relationships between the elasicities of substitution and the external economies of scale parameters (cf. Kucheryavyy et al., 2017). In this variant of our model, there are no markups and is clear that the market inefficiency is due only to upstream suppliers failing to internalize the positive productivity effects of their entry on downstream firms. This isomorphism will also prove to be useful in characterizing optimal trade policy in the open economy (see, in particular, section 3.2).

Although this vertical closed economy is generically inefficient, Proposition 1 highlights that efficiency is achieved when $\alpha = 1$ or $\alpha = 0$. The intuition for this is straightforward: in those cases, all labor is allocated to either the downstream sector (when $\alpha = 1$) or to the upstream sector (when

$\alpha = 0$), and because firm-level output is always efficient (see Proposition 1), there is no scope for a market inefficiency in those cases.

2.3 Optimal Policy

To analyze how a government can restore efficiency, suppose that it has the ability to provide production subsidies (or charge production taxes). We denote these taxes by s^d and s^u in the downstream and upstream sectors, respectively, and assume that subsidy proceeds are extracted from households (or tax revenue is rebated to households) in a lump-sum manner.

In Online Appendix A.3 we show that downstream subsidies s^d have no impact on the market allocation, while the following implementation result applies:

Proposition 2. The social planner can restore efficiency in the market equilibrium by subsidizing upstream production at a rate $(s^u)^* = 1/\theta$.

Notice that the subsidy corresponds to the reciprocal of the elasticity of substitution across inputs. As a result, this subsidy encourages entry of upstream suppliers, especially when the inputs they produce are relatively less substitutable. There are two potential (and non-exclusive) explanations for this result. First, the lower is θ, the larger is the market power of (and the markup charged by) input suppliers, and thus the larger the subsidy required to undo this double marginalization inefficiency. Second, the lower is θ, the larger are the productivity gains (i.e., variety gains) in the downstream sector associated with entry in the upstream sector. Because those gains are not internalized by input suppliers, the lower is θ, the larger is the required subsidy upstream (see also Online Appendix A.4). Notice that not only are downstream production subsidies redundant in our setting, but the optimal size of upstream subsidies is also independent of substitutability downstream (as governed by σ).

In Online Appendix A.5 we briefly develop two extensions of the simple model in this section. First, we allow the upstream sector to use the same bundle of inputs Q^u used in the final-good sector, while letting labor intensity upstream (denoted by β) differ from that downstream. Second, we outline a multi-stage extension of the model, in which the input bundle used in upstream production aggregates varieties from a yet more upstream sector, which in turns uses inputs from an even more upstream sector, and so on. In both extensions, we show that efficiency again calls for the use of subsidies in all input sectors, but not in the most downstream sector.[13]

Having explored the equilibrium and efficiency properties of a "Krugman" closed economy with an input sector, we next turn to exploring the open-economy implications of this framework. We are particularly interested in shedding light on the consequences of trade protection in both upstream and downstream sectors, and also on the optimal design of these trade policies. Beyond the double-marginalization and vertical-spillovers mechanism discussed above, the determination of

[13] Unlike in the work of Liu (2019), we do not find that optimal subsidies should necessarily be higher, the more upstream the sector. The reason is that, unlike in Liu's work, we solve for first-best subsidy policy: when the government can only set subsidies in one sector, the size of the subsidy is indeed higher, the more upstream the sector.

trade policies in our framework are shaped by both terms-of-trade considerations (as in Neoclassical Trade Theory), as well as relocation effects (as in New Trade Theory).

3 Open Economy Equilibrium: A Useful Isomorphism

We now consider a two-country extension of our two-sector model, which allows for (costly) international trade in both final goods and intermediate inputs. Throughout the rest of the paper, whenever the downstream sector uses labor and inputs (i.e., $\alpha \in (0,1)$), we focus on equilibria with incomplete specialization.

3.1 Environment with Internal Economies of Scale

There are now two countries (Home and Foreign), indexed by i or j (and sometimes by H and F), each populated by L_i consumers/workers. Trade is costly due to the presence of both iceberg trade costs and import tariffs. We denote by $\tau^d > 1$ and $\tau^u > 1$ the iceberg trade costs applying to final goods and to inputs, while we denote by t_i^d and t_i^u the tariffs set by country i on imports of final goods and intermediate inputs, respectively.

At times we will consider additional tax instruments, such as domestic production subsidies downstream and upstream (s_i^d and s_i^u), as well as export taxes on final goods and on inputs (v_i^d and v_i^u). We are, however, particularly interested in 'second-best' policies in which only import tariffs are available to the Home government. Concerning export taxes, it seems reasonable to ignore them initially since they are rarely used in practice and, in fact, some countries such as the United States explicitly ban them (see U.S. Constitution, Article 1, Section 9, Clause 5). As for production subsidies, we find it natural to study situations in which these instruments are not available to governments. The main reason for this is institutional in nature and relates to the fact that the WTO Agreement on Subsidies and Countervailing Measures ('SCM Agreement') significantly limits the scope of governments to use such instruments without punishment. We also note that, perhaps due to these institutional constraints, ruling out production subsidies is a widespread practice in theoretical analyses of trade policy determination in which production subsidies could well improve welfare, as exemplified in the work of Grossman and Helpman (1994) or Ossa (2011), among many others.

Denoting country $i \in \{H, F\}$ variables with i subindices, technologies upstream and downstream are now characterized by equations

$$f_i^u + x_i^u(\varpi) = A_i^u \ell_i^u(\varpi), \qquad \varpi \in [0, M_i^u], \quad i \in \{H, F\},$$

and

$$f_i^d + x_i^d(\omega) = A_i^d (\ell_i^d(\omega))^\alpha Q_i^u(\omega)^{1-\alpha}, \qquad \omega \in [0, M_i^d], \quad \alpha \in [0,1], \quad i \in \{H, F\}.$$

Because intermediate inputs are tradable, the bundle of inputs now includes both domestic and foreign input varieties:

$$Q_i^u(\omega) = \left[\sum_{j \in \{H,F\}} \left(\int_0^{M_j^u} q_{ji}^u(\varpi)^{\frac{\theta-1}{\theta}} d\varpi \right) \right]^{\frac{\theta}{\theta-1}}, \qquad \theta > 1, \quad i \in \{H, F\}.$$

The representative consumer in country i derives utility according to:

$$U_i = \left[\sum_{j \in \{H,F\}} \left(\int_0^{M_j^d} q_{ji}^d(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right) \right]^{\frac{\sigma}{\sigma-1}}, \qquad \sigma > 1, \quad i \in \{H, F\}. \tag{9}$$

where M_j^d is the endogenous measure of firms in country j. Implicit in these equations is the fact that because trade costs are all ad-valorem and preferences are CES, all firms in all sectors will find it profitable to sell in both markets. As in our closed-economy model, market structure in both sectors and both countries is characterized by monopolistic competition and free entry.

As mentioned above, the government imposes tariffs on imports of both final goods and inputs. Given symmetry across firms, the tariff revenue collected by the government is rebated to households via lump-sum transfers in an amount

$$R_i = \frac{t_i^d}{1+t_i^d} M_j^d p_{ji}^d q_{ji}^d + \frac{t_i^u}{1+t_i^u} M_i^d M_j^u p_{ji}^u q_{ji}^u + \frac{v_i^d}{1-v_i^d} M_i^d \tilde{p}_{ij}^d q_{ij}^d + \frac{v_i^u}{1-v_i^u} M_j^d M_i^u \tilde{p}_{ij}^u q_{ij}^u, \tag{10}$$

where p_{ji}^d and p_{ji}^u are the prices paid by consumers in i for final goods and by firms in i for inputs, and where $\tilde{p}_{ij}^d$ and $\tilde{p}_{ij}^u$ are the prices collected by producers in i when selling final goods and inputs in country j. When the government also levies production subsidies, this government balance condition needs to be modified in a straightforward manner.

3.2 An Isomorphic Competitive Economy with External Economies of Scale

It is not complicated to derive the equations characterizing the equilibrium of the above two-country economy as a function of the parameters of the model and the policy choices of a given country i, which we will associate with Home. In our analysis of optimal policy, we have however found it much more convenient to work with the equilibrium conditions of an isomorphic competitive economy with external rather than internal economies of scale.[14] In the remainder of this section, we develop such an isomorphic characterization, while we relegate a derivation of the equilibrium conditions of the above 'Krugman' economy with internal economies of scale to Online Appendix B.1.

Consider then a simpler economy in which there are only four goods: a Home final good, a Foreign final good, a Home intermediate input, and a Foreign intermediate input. Preferences in

[14]This isomorphism is inspired by the work of Kucheryavyy et al. (2017). We thank Steve Redding and Iván Werning for the helpful suggestion to pursue this direction.

country $i = \{H, F\}$ are given by

$$U\left(Q_{ii}^d, Q_{ji}^d\right) = \left(\left(Q_{ii}^d\right)^{\frac{\sigma-1}{\sigma}} + \left(Q_{ji}^d\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \tag{11}$$

where Q_{ii}^d is i's consumption of its local good, and Q_{ji}^d are imports by i of country j's good. As in our baseline model, the parameter σ governs the substitutability between the Home and Foreign goods.

The final good in each country is produced combining local labor (ℓ_i^d), the Home intermediate input (q_{ii}^u), and the Foreign intermediate input (q_{ji}^u). Technology is given by

$$x_i^d = \hat{A}_i^d F^d\left(\ell_i^d, q_{ii}^u, q_{ji}^u\right) = \hat{A}_i^d\left(\ell_i^d\right)^\alpha\left(\left(q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}(1-\alpha)},$$

where $\hat{A}_i^d$ is downstream productivity, where α determines the labor intensity of final-good production, and where θ governs the substitutability between the Home and the Foreign inputs. Downstream productivity $\hat{A}_i^d$ is in turn given by

$$\hat{A}_i^d = \bar{A}_i^d\left(F^d\left(L_i^d, Q_{ii}^u, Q_{ji}^u\right)\right)^{\gamma^d} = \bar{A}_i^d\left(\left(L_i^d\right)^\alpha\left(\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}}\right)^{\gamma^d}, \tag{12}$$

and is thus a function of country i's aggregate allocation of labor L_i^d to the downstream sector, the aggregate use of country i's intermediate input, and the aggregate use of country j's intermediate input. The parameter γ^d governs the degree of external economies of scale in the downstream sector, and is often referred to as the scale elasticity of this sector.

The intermediate input in each country is produced using local labor ℓ_i^u according to

$$x_i^u = \hat{A}_i^u F_i^u\left(\ell_i^u\right) = \hat{A}_i^u \ell_i^u,$$

where upstream productivity is also endogenous and given by

$$\hat{A}_i^u = \bar{A}_i^u\left(L_i^u\right)^{\gamma^u}, \tag{13}$$

where L_i^u is country i's aggregate allocation of labor to the upstream sector, and where the parameter γ^u governs the scale elasticity of the upstream sector.

We assume that the above technologies are available to a competitive fringe of producers in each country and sector. These producers take prices of all goods as given, and do not internalize the effects of their choices on the productivity terms $\hat{A}_i^d$ and $\hat{A}_i^u$. Given symmetry, it should be clear that in equilibrium, $\ell_i^d = L_i^d$, $\ell_i^u = L_i^u$, $q_{ii}^u = Q_{ii}^u$, and $q_{ji}^u = Q_{ji}^u$ for all firms. We can thus ignore lower-case variables hereafter.

The key conditions characterizing the decentralized market equilibrium in a given country i

consist of four resource constraints and four optimality conditions. The four resource constraints are (i) an aggregate labor market constraint

$$L_i = L_i^u + L_i^d, \tag{14}$$

(ii)-(iii) two equations equating output produced in each sector to its use (for domestic consumption or for export)

$$\hat{A}_i^u L_i^u = Q_{ii}^u + Q_{ij}^u; \tag{15}$$

$$\hat{A}_i^d \left(L_i^d\right)^\alpha \left(\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}} = Q_{ii}^d + Q_{ij}^d, \tag{16}$$

and (iv) a trade-balance condition

$$P_{ji}^d Q_{ji}^d + P_{ji}^u Q_{ji}^u = P_{ij}^d Q_{ij}^d + P_{ij}^u Q_{ij}^u. \tag{17}$$

In this last condition, the prices P_{ji}^d and P_{ji}^u reflect import prices collected by foreign exporters, so the domestic (country i) prices paid by the buyers of those goods are $(1+t_i^d)P_{ji}^d$ and $(1+t_i^u)P_{ji}^u$, respectively, where remember that t_i^d and t_i^u are the import tariffs set by country i. Similarly, the export prices P_{ji}^d and P_{ji}^u in the trade balance condition (17) correspond to the prices paid by foreign buyers, so the domestic (country i) price collected by the sellers of those goods are $(1-v_i^d)P_{ij}^d$ and $(1-v_i^u)P_{ij}^u$, respectively, where remember that v_i^d and v_i^u denote country i's downstream and upstream export taxes.

Turning to the four optimality conditions characterizing the decentralized equilibrium, the first two equations simply equate the marginal rate of substitution in final-good and intermediate-input consumption to the domestic (country i) relative price faced by the buyers of these goods, or

$$\frac{U_{Q_{ii}^d}\left(Q_{ii}^d, Q_{ji}^d\right)}{U_{Q_{ji}^d}\left(Q_{ii}^d, Q_{ji}^d\right)} = \frac{\left(1-v_i^d\right)}{\left(1+\tau_i^d\right)} \frac{P_{ij}^d}{P_{ji}^d}; \tag{18}$$

$$\frac{F_{Q_{ii}^u}^d\left(L_i^d, Q_{ii}^u, Q_{ji}^u\right)}{F_{Q_{ji}^u}^d\left(L_i^d, Q_{ii}^u, Q_{ji}^u\right)} = \frac{\left(1-v_i^u\right)}{\left(1+\tau_i^u\right)} \frac{P_{ij}^u}{P_{ji}^u}. \tag{19}$$

In these equations, subindices on the functions U and F^d denote partial derivatives of these functions with respect to the argument in the denominator. The next optimality condition ensures the equality between the benefits of exporting the domestic intermediate input to the benefits of using that amount of domestic inputs to produce an additional amount of the final good that is in turn exported:

$$\hat{A}_i^d F_{Q_{ii}^u}^d\left(L_i^d, Q_{ii}^u, Q_{ji}^u\right) = \frac{\left(1-v_i^u\right) P_{ij}^u}{\left(1-v_i^d\right) P_{ij}^d}. \tag{20}$$

The final efficiency condition equates the marginal product of labor in both sectors in terms of a common good (i.e., the final good)

$$F^d_{\ell^d_i}\left(L^d_i, Q^u_{ii}, Q^u_{ji}\right) = \hat{A}^u_i F^u_{L^u_i}\left(L^u_i\right) F^d_{Q^u_{ii}}\left(L^d_i, Q^u_{ii}, Q^u_{ji}\right). \tag{21}$$

Although we have developed equilibrium conditions (14) through (21) in a competitive model without meaningful firm-level decisions on entry, exporting, importing and pricing, we prove in Online Appendix B.2 that our baseline Krugman-style model with internal economies of scale and imperfect competition delivers the exact same set of equilibrium conditions for an appropriate choice of the primitive productivity terms $\bar{A}^d_i$ and $\bar{A}^u_i$ in equations (12) and (13), and as long as the scale elasticities γ^d and γ^u are set to $\gamma^d = 1/(\sigma - 1)$ and $\gamma^u = 1/(\theta - 1)$, respectively.

We summarize this discussion as follows (the proof is in Online Appendix B.2):

Proposition 3. The decentralized equilibrium of the two-country model in section 3.1 featuring internal scale economies, product differentiation and monopolistic competition can be reduced to a set of equations identical to equations (14) through (21) applying to the competitive model with external economies of scale developed in this section.

4 Optimal Trade Policy for a Small Open Economy with No Domestic Distortions

In this section, we consider the impact and optimal design of trade policies upstream and downstream for the special case in which the Home country is a small open economy (in a sense to be formalized), and in which the downstream sector does not employ labor (i.e., $\alpha = 0$). The main implication of the second assumption is that the allocation of labor across sectors is efficient (see Proposition 1) and independent of trade-policy choices. Furthermore, because firm-level output levels are also socially efficient (see Online Appendix B.1), trade policies will only seek to affect the measure of final-good producers entering in each country and the relative wage in the two countries. This allows us to compare our results more cleanly to those in the important contributions of Gros (1987), Venables (1987), and Ossa (2011) in "horizontal" models without an input sector. As in those frameworks, a combination of trade taxes applied to imports or exports will be sufficient to implement the first-best allocation, so domestic subsidies will be redundant instruments for the time being (although we consider them briefly later in this section).

Even for the case $\alpha = 0$, the characterization of the optimal choice of trade taxes is involved so, as mentioned above, we build intuition by considering the policy choices of a small open economy. We believe this case isolates the key mechanisms behind our results, and the formulas to be derived below provide an extremely good approximation for our quantitative findings in section 7.

We proceed in two steps. First, we consider the (unrestricted) set of trade policies that implement the first-best in our small open economy. Because such an optimal mix necessarily involves export taxes and these are often are not available to governments (e.g., they are forbidden in the US Constitution), we later consider the choice of second-best policies, when only import tariffs are allowed. As anticipated in section 3.2, we conduct our analysis based our isomorphic competitive economy featuring external economies of scale because this greatly simplifies the derivations.

4.1 First-Best Policies

To solve for the first-best policies, we closely follow the primal approach in Costinot et al. (2015).[15] More specifically, we first consider an environment in which a (fictitious) social planner directly controls consumption and output decisions, and we derive three key conditions characterizing the structure of the optimal allocation. We then compare these conditions to those from a decentralized market equilibrium in which the government has access to trade taxes (see section 3.2), and we finally show how the optimal allocation can be implemented through a simple combination of trade taxes. We later consider the case in which the government also has access to subsidies on domestic production or domestic consumption.

A. Optimal Allocation

Consider the problem of a Home social planner who seeks to maximize welfare in equation (11), subject to the labor-market constraint (14), the output-market constraints (15) and (16), and the trade balance condition (17). The planner is assumed to control domestic consumption (Q^d_{HH}, Q^u_{HH}), imports (Q^d_{FH}, Q^u_{FH}) and exports (Q^d_{HF}, Q^u_{HF}) of both final goods and intermediate inputs. Based on our recasting of our model in section 3.2, for the case $\alpha = 0$, this problem reduces to choosing $\left\{Q^d_{HH}, Q^d_{FH}, Q^d_{HF}, Q^u_{HH}, Q^u_{FH}, Q^u_{HF}\right\}$ to

$$\max \quad U\left(Q^d_{HH}, Q^d_{FH}\right) = \left(\left(Q^d_{HH}\right)^{\frac{\sigma-1}{\sigma}} + \left(Q^d_{FH}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

$$\begin{aligned} s.t. \quad & \hat{A}^u_H\left(L_H\right) L_H = Q^u_{HH} + Q^u_{HF} \\ & \hat{A}^d_H\left(F^d\left(Q^u_{HH}, Q^u_{FH}\right)\right) F^d\left(Q^u_{HH}, Q^u_{FH}\right) = Q^d_{HH} + Q^d_{HF} \\ & P^d_{FH} Q^d_{FH} + P^u_{FH} Q^u_{FH} = Q^d_{HF} (Q^d_{HF})^{-\frac{1}{\sigma}} P^d_{FF}\left(Q^d_{FF}\right)^{\frac{1}{\sigma}} + Q^u_{HF}\left(Q^u_{HF}\right)^{-\frac{1}{\theta}} P^u_{FF}\left(Q^u_{FF}\right)^{\frac{1}{\theta}}, \end{aligned}$$

where $\hat{A}^d_H\left(F^d\left(Q^u_{HH}, Q^u_{FH}\right)\right)$ and $\hat{A}^u_H\left(L_H\right)$ are given in (12) and (13) for $\alpha = 0$, respectively, and where

$$F^d\left(Q^u_{HH}, Q^u_{FH}\right) = \left(\left(Q^u_{HH}\right)^{\frac{\theta-1}{\theta}} + \left(Q^u_{FH}\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}. \tag{22}$$

The trade balance constraint in the above program requires a bit more explanation. First of all, notice that this equation is derived from (17), after substituting $P^d_{HF} = P^d_{FF}\left(Q^d_{HF}/Q^d_{FF}\right)^{-1/\sigma}$ and $P^u_{HF} = P^u_{FF}\left(Q^u_{HF}/Q^u_{FF}\right)^{-1/\theta}$, which correspond to Foreign's inverse demand for the Home final and intermediate-input goods, respectively.[16] Although we have assumed that Home is a small open economy, the fact that it produces *differentiated* final goods and *differentiated* intermediate

[15] See Costinot et al. (2020) and Kortum and Weisbach (2021) for other recent applications of this approach.

[16] These equations can in turn be derived based on the optimality conditions (18) and (19) applying when $i = F$ and $j = H$.

inputs still confers some market power to the Home government, in the sense that this government perceives a downward sloping demand for its goods. Second, it may seem non-standard to introduce prices in the constraint of a planner problem, but this is precisely where our assumption of Home being a small open economy is useful. More specifically, we assume that Home is small in the sense that its policy choices have *no impact* on Foreign's domestic prices P_{FF}^{d} and P_{FF}^{u}, or on the prices P_{FH}^{d} and P_{FH}^{u} (before import tariffs) collected by Foreign exporters. As a result, the Home government treats these prices as parameters in the planner problem above.

Working with the first-order conditions of this problem (see Online Appendix C.1), we characterize the first-best allocations via the three following conditions. First, on the consumption side, the Home social planner wants to equate the representative consumers' marginal rate of substitution with the *social* relative cost of domestic versus foreign goods

$$\frac{U_{Q_{HH}^{d}}\left(Q_{HH}^{d},Q_{FH}^{d}\right)}{U_{Q_{FH}^{d}}\left(Q_{HH}^{d},Q_{FH}^{d}\right)}=\frac{\frac{\sigma-1}{\sigma}P_{HF}^{d}}{P_{FH}^{d}}. \tag{23}$$

Note that the private cost of consuming domestic goods (which is equal to the opportunity cost P_{HF}^{d} of exporting these goods) exceeds its social cost. This wedge reflects a fairly standard rationale for terms-of-trade manipulation. In particular, for a given level of final-good production, an increase in domestic consumption Q_{HH}^{d} necessarily reduces exports, and this in turn raises Home's export prices and thus improves its terms of trade, even when Home is a small open economy (see Gros, 1987). Raising the private cost of imported goods or decreasing the private benefit of exporting goods by a factor $\sigma/(\sigma-1)$ restores the equality of the *relative* private and social cost of domestic and foreign goods.[17]

The second key efficiency condition is analogous to equation (23) and equates the marginal rate of substitution between domestic and foreign inputs in the production of final goods to the social relative cost of these inputs, or

$$\frac{F_{Q_{HH}^{u}}^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)}{F_{Q_{FH}^{u}}^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)}=\frac{\frac{\theta-1}{\theta}P_{HF}^{u}}{P_{FH}^{u}}. \tag{24}$$

The relative social cost of these inputs is again distinct from their relative private cost. The reason for this is analogous to the one in equation (23): the social cost of domestic inputs is lower than the private cost because the Home government perceives a downward sloping demand for its goods, and thus raising the private cost of imported inputs or decreasing the private benefit of exporting them is socially desirable.

[17] One may wonder whether an alternative interpretation of this result relates it to the fact that the markups charged by domestic firms on domestic consumers generate profits that remain in the Home country (or, more precisely, they lead to increased labor demand via firm entry seeking to dissipate those profits), while for imported goods, markups are collected by foreign firms, and thus the private and social marginal cost of those foreign goods coincide. Nevertheless, as we will further argue in section C.2, our preferred intuition is that this condition reflects market power in export markets driven by product differentiation, and has little to do with market structure or scale economies.

The final efficiency condition is given by

$$\left(1+\gamma^{d}\right)\hat{A}_{H}^{d}F_{Q_{HH}^{u}}^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)=\frac{\frac{\theta-1}{\theta}}{\frac{\sigma-1}{\sigma}}\frac{P_{HF}^{u}}{P_{HF}^{d}},\tag{25}$$

and equates the benefits of exporting domestic inputs to the benefits of using them to produce final goods that are in turn exported. This third equation takes into account both the productivity enhancing effects of boosting domestic production of final goods (the first term $1+\gamma^{d}$ on the left-hand side), as well as the relative potential for Home to exploit market power in its inputs versus final goods, which is mediated by the ratio $\left(\sigma/\left(\sigma-1\right)\right)/\left(\theta/\left(\theta-1\right)\right)$ on the right-hand-side of equation (25). Note that when σ and θ go to infinity, Home's market power disappears, and Home becomes a small open economy in the traditional sense, i.e, in the sense of being unable to affect its terms of trade through relative price effects.

B. First-Best Trade Policies

We are now ready to compare these optimal allocations to those from the decentralized equilibrium in which the government can set import tariffs or export taxes, as derived in section 3.2. It should be clear that there is a close connection between equations (18)–(20) applying in this decentralized equilibrium, and equations (23)–(25) characterizing the socially optimal allocations.[18]

There are two key differences between these two sets of equations. First, the market equilibrium conditions naturally incorporate the effect of taxes in shaping the private choices of individuals and firms. Second, these decentralized market equations do *not* incorporate the positive impact of downstream output expansion on productivity, or the positive effect of curtailing exports of final goods and of inputs on Home's terms of trade.

Simple comparison of these sets equations indicates that a combination of import tariffs and export taxes can achieve the first best as long as it satisfies:

$$1+t_{H}^{d} = \left(1+\gamma^{d}\right)\left(1+\bar{T}\right);\tag{26}$$

$$1+t_{H}^{u} = 1+\bar{T};\tag{27}$$

$$1-v_{H}^{d} = \frac{\sigma-1}{\sigma}\left(1+\gamma^{d}\right)\left(1+\bar{T}\right);\tag{28}$$

$$1-v_{H}^{u} = \frac{\theta-1}{\theta}\left(1+\bar{T}\right),\tag{29}$$

for any arbitrary constant such that $1+\bar{T}\geq0$.

A few comments are in order. First, note that the level of optimal import tariffs and export taxes is indeterminate in our setting. This a manifestation of Lerner's symmetry: optimal policies featuring a common ratio of gross import tariffs and export taxes (i.e., $\left(1+t_{H}^{s}\right)/\left(1-v_{H}^{s}\right)$ for $s=\{d,u\}$) deliver the exact same market allocations. Second, notice that the ratio of optimal gross import

[18] In section 3.2, we identified a fourth optimality condition – equation (21) – associated with the allocation of labor across sectors, but this condition is irrelevant when the downstream sector does not use labor (i.e. $\alpha=0$).

tariffs on final goods and on inputs, or the "tariff escalation wedge" *is* pinned down in our model and given by

$$\frac{1+t_H^d}{1+t_H^u} = 1+\gamma^d > 1.$$

Third, the optimal allocation *cannot* be achieved with only import tariffs, since implementing the first best requires distinct export taxes downstream and upstream.[19] Fourth, noting that our isomorphism applies only when $1+\gamma^d = \sigma/(\sigma-1)$, setting $\bar{T}=0$ minimizes the set of instruments necessary to achieve the first best. In such a case, the first-best policies involve only two instruments: a downstream import tariff at a level $t_H^d = \gamma^d = 1/(\sigma-1)$ and an upstream export tax v_H^u equal to $1/\theta$. In sum, we have derived the following result:

Proposition 4. When $\alpha=0$, the first-best allocation can be achieved with a combination of import and export trade taxes. Although, the levels of trade taxes are not uniquely pinned down, the tariff escalation wedge is necessarily given by $\left(1+t_H^d\right)/\left(1+t_H^u\right) = 1+\gamma^d = \sigma/(\sigma-1) > 1$. Furthermore, the first best can be achieved with just a downstream import tariff at a level t_H^d equal to $1/(\sigma-1)$ and an upstream export tax v_H^u equal to $1/\theta$.

Why do optimal policies involve higher import tariffs on final goods than on inputs? And why does the government choose to tax imports of final goods while taxing exports of inputs when using the minimum set of instruments? The key distinction between trade taxes on final goods and on inputs is as follows. A downstream import tariff or export tax shifts consumers' expenditures towards Home final-good varieties, thereby improving Home's terms of trade (Gros, 1987). The increased demand for Home's final goods also raises downstream productivity through increased entry. While this is similar to prior work on positive relocation effects (Venables, 1987; Ossa, 2011), when $\alpha=0$ the expansion of the downstream sector is due to increased input expenditures rather than a reallocation of labor. An upstream tariff also redirects Home expenditure towards Home inputs, which also improves its terms of trade, but raises domestic final-good producers' costs, which in turn reduces the downstream sector's production and thus its efficiency. As a result, the Home government has a disproportionate incentive to manipulate its terms of trade in the input sector via an export tax, which also shifts expenditure on inputs towards Home firms without raising their input costs. This is clear from equations (28) and (29), which show that the incentive to use upstream export taxes is magnified by a factor $1+\gamma^d$ relative to the incentive to use downstream export taxes. In other words, the returns to scale in downstream production govern the benefits from increasing the size of that sector. Because the relative size of sectoral import tariffs and export taxes is constrained by (18) and (19), this in turn manifests itself in the form of a lower import tariff upstream than downstream.

In sum, this special case of our model illustrates how tariff escalation is a feature of the optimal set of trade taxes in a vertical model in which intermediate inputs shape the cost faced by the downstream sector *and* the productivity of the downstream sector is endogenous to the size of this

[19]By Lerner symmetry, export taxes are redundant only if they can be set at the same level in all sectors.

sector. To further buttress the importance of this second aspect of our model, we next consider the nature of first-best policies in the absence of scale economies.

C. The Case of No Scale Economies

A key advantage of our isomorphic competitive economy with external economies of scale is that when $\gamma^d \to 0$, this economy converges to a competitive economy with no scale economies. With the expressions above, it is then straightforward to derive first-best trade policies in that case. Specifically, equations (26)–(29) now reduce to

$$\begin{aligned} 1+t_H^d &= 1+\bar{T}; \\ 1+t_H^u &= 1+\bar{T}; \\ 1-v_H^d &= \frac{\sigma-1}{\sigma}\left(1+\bar{T}\right); \\ 1-v_H^u &= \frac{\theta-1}{\theta}\left(1+\bar{T}\right), \end{aligned}$$

for any arbitrary constant such that $1+\bar{T} \geq 0$. It is then immediate that:

Proposition 5. When $\alpha = 0$, in the absence of scale economies, the first best can be attained with a combination of import and export taxes. Although, the levels of trade taxes are not uniquely pinned down, the tariff escalation wedge $\left(1+t_H^d\right)/\left(1+t_H^u\right)$ necessarily equals 1. Furthermore, the first best can be achieved with just a downstream export tax at a level v_H^d equal to $1/\sigma$ and an upstream export tax v_H^u equal to $1/\theta$.

This result shows that the emergence of tariff escalation is intimately tied to the existence of scale economies in the downstream sector. In their absence, we obtain a result analogous to that derived by Costinot et al. (2015) and by Beshkar and Lashkaripour (2020), namely that optimal trade policy involves uniform import tariffs across sectors (regardless of their differentiation or whether they are inputs or final goods) and differential export taxes that optimally exploit Home's market power.

We should briefly mention a caveat with the above argument. In particular, the isomorphism between our economies with internal economies of scale and with external economies imposes $\gamma^d \to 0 = 1/(\sigma-1)$. So it would appear that as $\gamma^d \to 0$, we must necessarily have $\sigma \to \infty$, which would imply that the Home economy has no market power in downstream markets. Note, however, that even in such a case, the model without scale economies would still not generate tariff escalation.

D. Generalizations

In this section we briefly comment on a few generalizations of our result in Proposition 4. The goal is not only to illustrate the robustness of our results, but also to solidify the intuition that we have favored in interpreting these results.

A first observation is that, in deriving our main results, we have not invoked the fact that Home preferences $U_H\left(Q^d_{HH}, Q^d_{FH}\right)$ in (11) or that the aggregator $F^d\left(Q^u_{HH}, Q^u_{FH}\right)$ of inputs in (22) are CES aggregators governed by σ and θ, respectively. More specifically, these aggregators could be governed by different parameters, or they could be more general (though twice-continuously differentiable) functions. In other words, the parameters σ and θ in the first-best policies are solely related to parameters governing preferences and technology *in Foreign*, not at Home; only the scale elasticity parameter γ^d in these formulas is associated with features of the Home economy. This reinforces our interpretation that the appearance of σ and θ in the formulas above is associated with standard terms-of-trade-manipulation incentives, rather than with the markups faced by domestic buyers. In other words, even when $\sigma \to \infty$ and $\theta \to \infty$, so Home ceases to have any market power in exports, our model continues to rationalize tariff escalation as long as $\gamma^d > 0$.

It is tempting to argue that the above discussion also implies that our results are robust to non-CES functional forms for $U_H\left(Q^d_{HH}, Q^d_{FH}\right)$ and $F^d\left(Q^u_{HH}, Q^u_{FH}\right)$, but it is important to remember that a CES functional form is crucial for the isomorphism we have developed in section 3.2. Hence, the first-best trade policies of a Krugman-style small open economy featuring monopolistic competition, CES preferences, and imperfect competition would be a function of the degree of substitutability σ_H in Home preferences because remember that $\gamma^d = 1/\left(\sigma_H - 1\right)$ in that model (though, in principle, we could have $\sigma_H \neq \sigma$). We demonstrate this explicitly in Online Appendix C.2.

E. Alternative First-Best Implementations

So far we have focused on implementations of the first-best allocations that involve the use of only trade taxes. A natural question is whether our rationale for tariff escalation relies on ruling out the use of *domestic* tax instruments. At some level, it should not be surprising that expanding the set of instruments available to the Home government will affect the structure of trade taxes. For instance, it is well known that, in some settings, it is straightforward to replicate the real effects of an import tariff with a combination of consumption taxes and production subsidies. Still, if import tariffs on inputs are unappealing because they reduce entry and industry-level productivity in the downstream sector, one may wonder whether tariff escalation is still necessary to achieve the first best once domestic subsidies are available to the government.

In Online Appendix C.3, we explore the structure of first-best policies when the set of available instruments includes domestic production subsidies, domestic consumption subsidies, or domestic production/consumption subsidies that only apply to domestic transactions. We relegate the mathematical details to the Online Appendix, but the main takeaways are as follows. First, the only way to implement the first-best using *only* two domestic instruments (analogously to the two trade policy instruments in Proposition 4) is via the use of discriminatory consumption subsidies applying only to domestic purchases of final goods and of inputs. More specifically, the first best can be achieved via a subsidy to the consumption of domestic intermediate inputs at a rate equal to $s^u_{HH} = 1/\theta$ (which coincides with the level that implements the first best in the closed economy) and a subsidy to the consumption of domestic final goods equal to $s^d_{HH} = 1/\sigma$. These subsidies serve the role of selectively boosting the size of the downstream domestic sector, while constraining exports to optimally manipulate the terms of trade. In addition, an import tariff downstream is a perfect substitute for a discriminatory consumption subsidy downstream, while an export tax upstream is a perfect substitute for a discriminatory consumption subsidy upstream. It then follows that the first best can also be achieved with a combination of a subsidy in one sector and a trade instrument in the other sector. Whether tariff escalation remains a feature of the first-best policies is sensitive to which precise instruments are used in the implementation.

When discriminatory consumption subsidies are not available – as they typically are not – the first best *cannot* be achieved with just two instruments, unless these two instruments are trade policy instruments, as in our implementation in Proposition 4. If the government chooses to rely on non-discriminatory production or consumption subsidies, it can achieve the first best combining an appropriate level of these non-discriminatory subsidies with either export taxes or import tariffs in both sectors. In particular, production subsidies require export taxes to offset the portion of the subsidy that accrues to foreign, while consumption subsidies require import tariffs to offset the subsidy on domestic firms' and consumers' purchases of foreign goods. The implied tariff escalation level is naturally sensitive to which precise instruments are used in the implementation of the first best. To reiterate, however, any implementation of the first best involving a domestic subsidy requires at least three tax instruments, rather than just two, as in Proposition 4. In addition, it is the use of those redundant subsidies themselves that motivates the use of tariffs that do not feature escalation, rather than the underlying structure of economy.

4.2 Second-Best Import Tariffs

We now consider an environment in which the only policies available to the Home government are import tariffs on final goods and on inputs. As shown in the previous sections, this pair of tariffs is not sufficient to achieve the first-best allocation, so a natural question is whether second-best import tariffs will continue to feature tariff escalation.

A. Second-Best Import Tariffs with Scale Economies

For the case of a small open economy, the primal approach developed in Costinot et al. (2015) is again very useful to characterize the second-best allocation and how they can be implemented with only import tariffs. In particular, the second-best optimal allocation seeks to solve the same problem laid out in section 4.1 expanded to include an additional constraint. Intuitively, the Home planner can always ensure that the optimality conditions (23) and (24) are satisfied via an appropriate choice of import tariffs. On the other hand, absent export taxes ($v_H^d = v_H^d = 0$), equation (20) reduces to

$$\hat{A}_H^d F_{Q_{HH}^u}^d \left(Q_{HH}^u, Q_{FH}^u\right) = \frac{P_{HF}^d}{P_{HF}^u} = \frac{\left(Q_{HF}^u\right)^{-\frac{1}{\theta}} P_{FF}^u \left(Q_{FF}^u\right)^{\frac{1}{\theta}}}{\left(Q_{HF}^d\right)^{-\frac{1}{\sigma}} P_{FF}^d \left(Q_{FF}^d\right)^{\frac{1}{\sigma}}}, \tag{30}$$

which cannot be affected directly via import tariffs. As in the first-best case, the Home government will internalize (but private agents will fail to internalize) the fact that the ratio of export prices is shaped by Home's relative export supply of inputs and final goods.

In Online Appendix C.4, we work with the first-order conditions of the planner problem, compare them those applying to a decentralized market equilibrium with only import tariffs – which corresponds to equations (18)–(20) setting $v_H^d = v_H^d = 0$ – and establish that:

Proposition 6. When $\alpha = 0$, the second-best optimal combination of import tariffs involves an import tariff on final goods t_H^d higher than $1/(\sigma - 1)$ and a tariff escalation wedge larger than the first-best one, so $(1 + t_H^d)/(1 + t_H^u) > 1 + \gamma^d = \sigma/(\sigma - 1) > 1$.

To understand the intuition behind this result, it is useful to remember the role that export taxes served in the set of first-best trade policies. First, export taxes were key in engineering differential terms-of-trade manipulation in final-good versus input markets. Second upstream export taxes also provided a tool to manipulate the terms of trade for inputs in a less distortionary way (for the size and productivity of the downstream sector) than upstream import tariffs. In the absence of export taxes, the Home government will find it optimal to use import tariffs on inputs to manipulate the terms of trade (since it cannot rely on export taxes to do so), but it is also intuitive that such an incentive is attenuated relative to the incentive to use import tariffs on final goods. Downstream import tariffs redirect demand toward the Home market, thus generating positive spillover effects and also curtailing exports and thus improving the terms of trade for final goods. Upstream import tariffs have similar effects on the upstream market, but at the same time, they increase costs and reduce the size (and thus the productivity) of the Home downstream sector. As a result of these forces, even in the second-best policies, the tariff escalation wedge $(1 + t_H^d)/(1 + t_H^u)$ remains above one, and is in fact higher than in the first-best, as a higher downstream tariff is now necessary to compensate for the negative impact of upstream import tariffs on entry in the Home downstream sector.

B. Second-Best Import Tariffs with No Scale Economies

It is also instructive to characterize second-best import tariffs in the absence of scale effects. Do these also necessarily feature tariff escalation? Characterizing these policies is again quite straightforward, since we need only consider the case when $\gamma^d \to 0$ in our competitive economy with external scale economies. In Online Appendix C.4, we prove the following result:

Proposition 7. In the absence of scale economies, the second-best optimal combination of import tariffs involves tariff escalation (i.e., $\left(1 + t_H^d\right)/(1 + t_H^u) > 1$) if and only if $\sigma > \theta$.

To understand this result it is useful to first focus on the case $\sigma = \theta$. In a competitive Ricardian model, as long as optimal export taxes are common across sectors (i.e., $\sigma = \theta$ in our setting), the first-best can be implemented via either export taxes or import tariffs (Costinot et al., 2015; Beshkar and Lashkaripour, 2020). As a result, second-best import tariffs will achieve the first-best allocations in such a case, and they will be necessarily equal across sectors (regardless of their upstreamness). In such a case, this competitive model with no scale effects deliver no tariff escalation.

Starting from this benchmark, when $\sigma \neq \theta$, the first-best can no longer be implemented with only import tariffs. Beshkar and Lashkaripour (2017) show that in a horizontal economy without vertical links across sectors, second-best import tariffs continue to be common across countries. In the presence of vertical links, Beshkar and Lashkaripour (2020) instead point out that import tariffs can partly mimic the effects export taxes by raising the relative price of downstream sectors. If the planner would like to set a higher export tax in one sector relative to another sector, it can adjust the relative size of the second-best import tariff on inputs to achieve the desired differential terms-of-trade manipulation. When $\sigma < \theta$, the desired export tax is higher downstream, so the government will actually implement tariffs featuring higher tariffs upstream (i.e., tariff de-escalation). Conversely, when $\sigma > \theta$, the planer would prefer a lower export tax downstream, which can be partly achieved by setting a lower tariff on intermediate inputs (i.e., tariff escalation), since this reduces the relative price of exports of the downstream good.[20]

5 Optimal Trade Policy for a Small Open Economy with Domestic Distortions

In this section, we consider environments in which final-good production uses both inputs and labor in production. As a result, the final-good and intermediate input sectors compete for labor, and as we showed in section 2, the intersectoral allocation of labor in the decentralized equilibrium will be inefficient, and will feature too little labor allocated to the upstream sector. This labor misallocation naturally has ramifications for the set of first-best – as trade taxes will no longer be sufficient to achieve the optimal allocation – and also for the second-best import tariffs, as the levels of these instruments will now be set partly with the goal to alleviate this domestic friction.

5.1 First-Best Policies

We begin by studying the optimal structure of first-best policies. As in section 4, we follow the primal approach in Costinot et al. (2015) and first characterize the optimal allocation, and later show how to implement it via trade taxes and domestic instruments. Because many of the derivations are analogous to those in section 4, we relegate many details to Appendix D.1.

Consider first the determination of the optimal allocation. This problem is analogous to that solved in section 4.1, except that (i) the planner now also controls the allocation of labor across sectors subject to a labor-market constraint, and that (ii) productivity in *both* sectors is now endogenous and shaped by the allocation of labor to each sector. More precisely, the planner chooses

[20] Interestingly, Proposition 7 continues to hold unaltered when $\alpha > 0$, which is the reason why its statement does not impose the proviso $\alpha = 0$.

$\left\{L_H^u, L_H^d, Q_{HH}^d, Q_{FH}^d, Q_{HF}^d, Q_{HH}^u, Q_{FH}^u, Q_{HF}^u\right\}$ to

$$\max \quad U\left(Q_{HH}^d, Q_{FH}^d\right) = \left(\left(Q_{HH}^d\right)^{\frac{\sigma-1}{\sigma}} + \left(Q_{FH}^d\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

$$\begin{aligned} s.t. \quad & L_H^u + L_H^d = L_H \\ & \hat{A}_H^u\left(L_H^u\right)L_H^u = Q_{HH}^u + Q_{HF}^u \\ & \hat{A}_H^d\left(F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)\right)F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) = Q_{HH}^d + Q_{HF}^d \\ & P_{FH}^d Q_{FH}^d + P_{FH}^u Q_{FH}^u = Q_{HF}^d (Q_{HF}^d)^{-\frac{1}{\sigma}} P_{FF}^d \left(Q_{FF}^d\right)^{\frac{1}{\sigma}} + Q_{HF}^u \left(Q_{HF}^u\right)^{-\frac{1}{\theta}} P_{FF}^u \left(Q_{FF}^u\right)^{\frac{1}{\theta}}, \end{aligned}$$

where $\hat{A}_H^d\left(F^d\left(L_H^d, Q_{HH}^u, Q_{HF}^u\right)\right)$ and $\hat{A}_H^u\left(L_H^u\right)$ are given in (12) and (13), respectively, and where

$$F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) = \left(L_i^d\right)^{\alpha}\left(\left(Q_{HH}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{FH}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}}.$$

As we show in Online Appendix D.1, manipulating the first-order conditions of this problem produces three optimality conditions identical to those in equations (23), (24), and (25), except for the fact that L_H^d now appears as an argument of the partial derivative terms associated with the function $F^d(\cdot)$ in equations (24), and (25). More substantively, the optimal allocation now also includes a fourth optimality condition,

$$F_{L_H^d}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) = \left(1+\gamma^u\right)\hat{A}^u\left(L_H^u\right)F_{Q_{HH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right), \tag{31}$$

which equates the social value of the marginal product of labor in both sectors in terms of a common good (i.e., the final good). More specifically, the left-hand-side includes terms associated with the social marginal product of directly allocating labor to the production of final goods, while the right-hand-side contains terms related to the social marginal product of allocating labor to the upstream sector, and then using the resulting intermediate inputs to increase the production of final goods.[21]

We next compare these optimal allocations to those applying in a decentralized equilibrium in which the government can set taxes or subsidies on all transactions. In section 4.1, and in particular, in equations (18)–(20), we show how trade taxes affect the market-equilibrium analogues of conditions (23)–(25). Because condition (31) is an internal optimality condition involving only domestic transactions, trade taxes cannot possibly affect it. In fact, the *only* type of taxes that can affect it are taxes or subsidies affecting the production or consumption of domestic inputs. In particular, denoting by s_H^u the subsidy applying to transactions of intermediate inputs, the

[21]The left-hand-side and right-hand-side of (31) do not *exactly* capture the social marginal return to labor because we have cancelled terms capturing the endogenous increase in productivity associated with the expansion of the final-good sector (see Appendix D.1).

market-equilibrium analogue of equation (31) is

$$F^d_{L^d_H}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) = \frac{1}{1-s^u_H}\hat{A}^u\left(L^u_H\right)F^d_{Q^u_{HH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right). \tag{32}$$

Comparing equations (31) and (32) – it is clear that the implementation of the optimal allocation necessarily requires a subsidy equal

$$s^u_H = \frac{\gamma^u}{1+\gamma^u} = \frac{1}{\theta},$$

which coincides with the optimal subsidy in the closed-economy version of our model (see Proposition 2).

How does the use of this upstream domestic subsidy affect the nature of the other first-best policies? Analogously to our analysis in section 4, we first focus on the case in which the government minimizes the use of non-trade taxes. Because downstream domestic subsidies are redundant instruments in our model, we initially rule them out (though we will consider them below). If the government has access to an upstream production subsidy s^u_H, we show in Appendix D.2 that the first-best instruments implementing the social optimality conditions (23)–(25) must satisfy the exact same conditions (26)–(29) as in the case $\alpha = 0$, namely

$$\begin{aligned}
1+t^d_H &= \left(1+\gamma^d\right)\left(1+\bar{T}\right);\\
1+t^u_H &= 1+\bar{T};\\
1-v^d_H &= \frac{\sigma-1}{\sigma}\left(1+\gamma^d\right)\left(1+\bar{T}\right);\\
1-v^u_H &= \frac{\theta-1}{\theta}\left(1+\bar{T}\right),
\end{aligned}$$

for any arbitrary constant such that $1+\bar{T} \geq 0$. As a result, we can conclude that:

Proposition 8. When $\alpha > 0$, the first-best allocation can be achieved with a production subsidy for inputs, and (at least two) trade taxes associated with a tariff escalation wedge $(1+t^d_H)/(1+t^u_H) = 1+\gamma^d = \sigma/(\sigma-1) > 1$. Furthermore, the first best can be achieved with just a downstream production subsidy s^u_H equal to $1/\theta$, a downstream import tariff at a level t^d_H equal to $1/(\sigma-1)$, and an upstream export tax v^u_H equal to $1/\theta$.

In words, once the domestic distortion identified in the closed-economy version of our model is corrected via a downstream production subsidy, the first-best can then attained via trade instruments in a manner analogous to that in the case of no domestic distortions ($\alpha = 0$). As a result, first-best policies continue to be consistent with trade policies featuring tariff escalation for reasons analogous to those laid out in section 4.

Alternative Implementations In Online Appendix D.2, we study the robustness of Proposition 8 to the use of a wider set of instruments, other than upstream production subsidies and trade taxes.

The results are largely analogous to those obtained for the case $\alpha = 0$. First, if the government has access to *discriminatory* upstream subsidies (applying only to domestic transactions involving intermediate inputs), then the first best can actually be achieved with *only* two instruments: this upstream discriminatory subsidy at a level $1/\theta$ and a downstream import tariff equal to $\sigma/(\sigma - 1)$, again implying a tariff escalation wedge higher than one. If in addition the government has access to a downstream discriminatory subsidy, this instrument proves to be a perfect substitutes for an import tariff. Thus, the first best can also be achieved with these two discriminatory subsidies. Although, downstream import tariffs and downstream discriminatory subsidies are perfect substitutes, an upstream export tax and an upstream domestic discriminatory subsidy are *no longer* perfect substitutes when $\alpha > 0$. Only the latter instrument can ensure that condition (31) is satisfied in the decentralized equilibrium.

In Online Appendix D.2, we also study the situations in which discriminatory domestic subsidies are not available, but non-discriminatory production or consumption subsidies are. In those cases, the first-best always requires the use of a mix of domestic subsidies and trade taxes, but the set of required instruments is always minimized when downstream domestic subsidies are *not* used. When only upstream production subsidies and trade taxes are used, we have argued above that the first-best policies continue to feature a tariff escalation wedge equal to $\sigma/(\sigma - 1)$, while when upstream consumption subsidies are used, the implied tariff escalation in the first-best policies depends on the relative size of σ and θ.[22]

The Case of No Scale Economies As in section 4, it is straightforward to use our analysis to shed light on optimal policy in the absence of scale economies. This simply amounts to setting $\gamma^d = \gamma^u = 0$. In such a case, it is straightforward to verify that Proposition 4 continues to apply: although, the levels of first-best trade taxes are not uniquely pinned down, the tariff escalation wedge $\left(1 + t_H^d\right)/\left(1 + t_H^u\right)$ necessarily equals one, and the first-best can be achieved with just two export taxes.

5.2 Second-Best Trade Policies

We now revert back to a more realistic case in which the only policies available to the Home government are import tariffs on final goods and on inputs. All implementations of the first best in the previous section involved at least one export tax or at least one domestic instrument, so it follows that the first best cannot be achieved with only import tariffs. Furthermore, in the absence of other instruments, import tariffs will seek to mimic the role that those ruled out instruments played in the first best implementation.

As in section 4, it is straightforward to see that when upstream export taxes are ruled out, the

[22] The use of downstream consumption subsidies has no bearing on the decentralized equilibrium, while the use of downstream production subsidies is more material for our finding of tariff escalation, precisely because as noted above, a downstream production subsidy boosts domestic production in a way similar to a downstream import tariff. Nevertheless, a downstream production subsidy needs to be complemented by a downstream export tax, and thus the first-best cannot be attained with less than four instruments.

Home government will seek to manipulate its terms of trade via upstream import tariffs. Second-best policies thus involve positive upstream import tariffs. Nevertheless, and as discussed in section 4, this force is not sufficient on its own to undo the desirability of tariff escalation, as formalized in Proposition 6. When $\alpha > 0$, however, the Home government would like to use an upstream subsidy to reallocate labor from the downstream to upstream sector, thereby increasing productivity not only upstream but also downstream. Because an upstream tariff partially mimics the effect of the missing subsidy by shifting final-good producers' input expenditures towards domestic varieties, this provides an additional welfare motive for an input tariff in the second-best, particularly when the downstream sector's labor share is high. At the same time, as the downstream labor share rises, inputs are relatively less important for final-good output.

Despite these counterbalancing forces, we have failed to find a single numerical simulation in which the second-best combination of import tariffs upstream and downstream do not satisfy $t_H^d > t_H^u$. This motivates the following conjecture:

Conjecture 1. Even when $\alpha > 0$, the second-best optimal combination of import tariffs is associated with a tariff escalation wedge larger than one, i.e., $(1 + t_H^d)/(1 + t_H^u) > 1$.

In Online Appendix D.3, we attempt to evaluate this conjecture analytically, following the same approach as in section 4.2. Specifically, we note that the Home planner can always ensure that the optimality conditions (23) and (24) are satisfied via an appropriate choice of import tariffs, but conditions (20) with $v_H^d = v_H^d = 0$ and (21) become additional constraints in the planner problem. The complexity of the analysis has however precluded an analytical proof to date.

The special case of our model with no scale economies upstream (i.e., $\gamma^u = 0$) proves to be much more tractable. In particular, we show that the result in Proposition 6 continues to hold even when $\alpha > 0$, and thus import tariffs result in a tariff escalation wedge larger than the first-best one, or $(1 + t_H^d)/(1 + t_H^u) > 1 + \gamma^d = \sigma/(\sigma - 1) > 1$. This highlights again the role of downstream scale economies in generating tariff escalation. Indeed, when we further set $\gamma^d = 0$, so the model features constant returns to scale in both sectors, we show in Online Appendix D.3 that tariff escalation is a feature of optimal tariffs only when $\sigma > \theta$, just as in our previous Proposition 7 for the case $\alpha = 0$.

6 First-Order Welfare Effects of Small Import Tariffs

To build further intuition for our quantitative results, particularly those related to our robust finding of tariff escalation in the second-best set of policies, we next study the first-order welfare effects of *small tariffs* levied by the 'Home' government on imported final goods or imported inputs. In doing so, we revert back to an environment with internal economies of scale, product variety and monopolistic competition. Relative to our above analysis in the isomorphic competitive economy with external economies of scale, the tools developed in this section will allow us to better flesh out the effects of import tariffs on firm relocation. In the rest of this section, we allow the final-good sector to use labor in production (i.e., $\alpha > 0$), and we no longer restrict Home to be a small open economy.

Because welfare at Home corresponds to the representative household's real income, we have

$$U_H = \frac{w_H L_H + R_H}{P_H^d},$$

where R_H is tariff revenue in equation (10), and where P_H^d is the ideal price index at Home.

We are interested in the change in Home's welfare associated with a change in the tariff schedule $\{t_H^d, t_H^u\}$ starting from an equilibrium with zero tariffs. For simplicity, and without loss of generality, we set the Home wage to be the numéraire, so we can focus on the effect of tariffs on tariff revenue and the price index. The change in Home's welfare, dU_H, around $t_H^d = 0$ and $t_H^u = 0$ (and thus $R_H = 0$), can then be written as:

$$\frac{dU_H}{U_H} = \left[-\frac{dP_H^d}{P_H^d} + \frac{dR_H}{w_H L_H} \right], \tag{33}$$

with

$$\frac{dR_H}{w_H L_H} = b_F^H \times dt_H^d + \lambda_H^d \times \Omega_{FH} \times dt_H^u, \tag{34}$$

where $b_F^H \equiv \frac{M_F^d p_{FH}^d q_{FH}^d}{w_H L_H}$ is the share of Home income spent on foreign varieties, $\lambda_H^d \equiv \frac{M_H^d p_H^d x_H^d}{w_H L_H}$ is the ratio of domestic final-good revenue to national income (with $R_H = 0$) in country H, and $\Omega_{FH} \equiv \frac{M_F^u M_H^d p_{FH}^u q_{FH}^u}{M_H^d p_H^d x_H^d}$ is the share of Home final-good revenue spent on intermediate input varieties from F.

Consider next the change in Home's ideal price index. Given the formula for this price index – see equations (B.6) and (B.8) in Online Appendix B.1 – and given firm symmetry, we have:

$$\frac{dP_H^d}{P_H^d} = b_H^H \times \left(\frac{1}{1-\sigma} \frac{dM_H^d}{M_H^d} + \frac{dp_{HH}^d}{p_{HH}^d} \right) + b_F^H \times \left(\frac{dM_F^d}{M_F^d} \frac{1}{1-\sigma} + \frac{dp_{FH}^d}{p_{FH}^d} + dt_H^d \right). \tag{35}$$

The ideal (downstream) price index changes because in equilibrium the total measure of firms, in both Home and Foreign, responds to the change in tariff. At the same time, the change in relative prices also affects the price charged by downstream producers. The size of each factor's contribution to the change in the price index depends on the importance of foreign and domestic goods in the consumption basket, b_j^H. The change in the unit price of downstream goods is given by:

$$\frac{dp_{ii}^d}{p_{ii}^d} = \alpha \frac{dw_i}{w_i} + (1-\alpha) \frac{dP_i^u}{P_i^u}, \tag{36}$$

with

$$(1-\alpha) \frac{dP_i^u}{P_i^u} = \left(\frac{dM_i^u}{M_i^u} \frac{1}{1-\theta} + \frac{dp_{ii}^u}{p_{ii}^u} \right) \Omega_{i,i} + \left(\frac{dM_j^u}{M_j^u} \frac{1}{1-\theta} + \frac{dp_{ji}^u}{p_{ji}^u} + dt_i^u \right) \Omega_{ji}. \tag{37}$$

This latter equation captures the change in the upstream price index in each country, which is in turn shaped by the change in the measure of upstream firms in each country, the change in the price of individual input varieties, and the relative importance of domestic and foreign inputs in production, as captured by the terms Ω_{ii} and Ω_{ji}. Since we have set Home wages as the numéraire, we have $\frac{dw_H}{w_H} = 0$. Also, since we hold iceberg trade costs fixed in this exercise, we have $\frac{dp^d_{ji}}{p^d_{ji}} = \frac{dp^d_{ii}}{p^d_{ii}}$. Finally, since upstream goods only use labor in production, we have $\frac{dp^u_{FF}}{p^u_{FF}} = \frac{dp^u_{FH}}{p^u_{FH}} = \frac{dw_F}{w_F}$.

Putting all the pieces together – that is, combining equations (33)-(37) – we finally obtain the following expression for the first-order effect of tariffs on Home welfare:

$$
\begin{aligned}
\frac{dU_H}{U_H} = & -\left(b^H_H \Omega_{FH} + b^H_F \left(\Omega_{FF} + \alpha\right)\right) \frac{dw_F}{w_F} \\
& + \left(\frac{b^H_H \Omega_{HH} + b^H_F \Omega_{HF}}{\theta - 1}\right) \frac{dM^u_H}{M^u_H} + \left(\frac{b^H_H \Omega_{FH} + b^H_F \Omega_{FF}}{\theta - 1}\right) \frac{dM^u_F}{M^u_F} \\
& + \left(\frac{b^H_H}{\sigma - 1}\right) \frac{dM^d_H}{M^d_H} + \left(\frac{b^H_F}{\sigma - 1}\right) \frac{dM^d_F}{M^d_F} \\
& + \left(\lambda^d_H - b^H_H\right) \Omega_{FH} (dt^u_H) \mathbb{I}_{\{t = t^u\}}.
\end{aligned} \tag{38}
$$

This expression contains six terms.[23] The first one captures 'factorial' *terms-of-trade* benefits from raising tariffs, which in our Ricardian model operate via changes in relative wages (or foreign wages, given our choice of numéraire). The next four terms capture *relocation effects* due to changes in the masses of domestic and foreign firms in both the upstream and downstream sectors. All terms enter positively, reflecting the positive effect of increased varieties upstream and downstream on welfare, but it should be clear that general-equilibrium constraints will preclude all these measures of firms from increasing in reaction to Home import tariffs. How each of these relocation effects influence welfare is in turn given by the relative importance of these four types of firms in the purchases of Home consumers and Home firms.

To provide intuition for the quantitative results to come, notice that due to home bias, we typically have $b^H_H > b^F_H$ and also $b^H_H > b^H_F$. Furthermore, $\Omega_{HH} \leq 1 - \alpha < 1$, and b^F_H will be small unless Home is a large economy. As a result, it will typically be the case that $b^H_H > b^H_H \Omega_{HH} + b^H_F \Omega_{HF}$. This carries two significant implications. First, a given percentage increase in the measure of domestic downstream firms (dM^d_H/M^d_H) has a larger impact on Home welfare than the same percentage increase in the measure of foreign downstream firms (dM^d_F/M^d_F). Second, a given percentage increase in the measure of domestic downstream firms (dM^d_H/M^d_H) has a larger impact on Home welfare than the same percentage increase in the measure of domestic upstream firms (dM^u_H/M^u_H). This suggests that, on account of relocation effects, (i) the Home government will have an incentive to levy import tariffs downstream to attract the entry of final-good producers into its economy – as highlighted in the work of Venables (1987) and Ossa (2011) –, and (ii) although such an incentive also exists with regard to entry of upstream firms, the net welfare effects of input producers' entry are smaller.

The sixth and final term in equation (38) is more subtle and relates to a key term identified in

[23]Note that while the downstream tariff has direct effects on the price index and on tariff revenues, these two effects are exactly offsetting, so that the only net effects on welfare operate indirectly through equilibrium variables.

the work of Beshkar and Lashkaripour (2020). More specifically, notice that $\lambda_H^d - b_H^H$ represents the value of exported downstream goods as a share of Home's GDP. This last term then captures the extent to which an input tariff is passed on to foreign consumers, thereby mimicking an export tax, which is part of the unconstrained set of optimal trade taxes in our environment with product differentiation and Ricardian technologies (Costinot et al., 2015; Beshkar and Lashkaripour, 2020). For this same reason, this last term *only* applies to changes in input tariffs.

Because changes in tariffs do not enter the other terms in equation (38) explicitly, it would appear that (small) intermediate-input import tariffs increase welfare by more than (small) final-good tariffs on account of this last extra term. Nevertheless, we have already indicated above that final-good and input tariffs generate differential effects on relative wages (dw_F/w_F) and on relocation effects (dM_i^s/M_i^s for $s = d, u$), and we will show in the next section that these channels are quantitatively dominant. More precisely, and anticipating the quantitative results to come, small final-good tariffs appear to generate larger welfare gains than small input tariffs, and relocation effects seem to be the quantitatively dominant force in shaping this differential response. When computing optimal tariffs for final goods and for inputs, we will find that optimal final-good tariffs are consistently larger than optimal input tariffs across a wide range of parameter values.

7 Quantitative Results

In this section, we perform a quantitative evaluation of the effects of import and final-good tariffs on welfare, and we solve for the social-welfare maximizing levels of these tariffs. We carry out this analysis by mapping world data to our two-country model, interpreting the Home country as the United States, and the Foreign country as the Rest of the World (RoW, hereafter). Our main result in this section is that tariff escalation appears to be a robust feature of the structure of optimal import tariffs. More specifically, when the Home country only has access to import tariffs, we find that import tariffs on intermediate inputs are systematically lower than tariffs on final goods. Our finding of a tariff escalation wedge is robust to the inclusion of optimal export taxes and of domestic subsidies, as long as downstream subsidies are not part of the policies used to implement the first best.

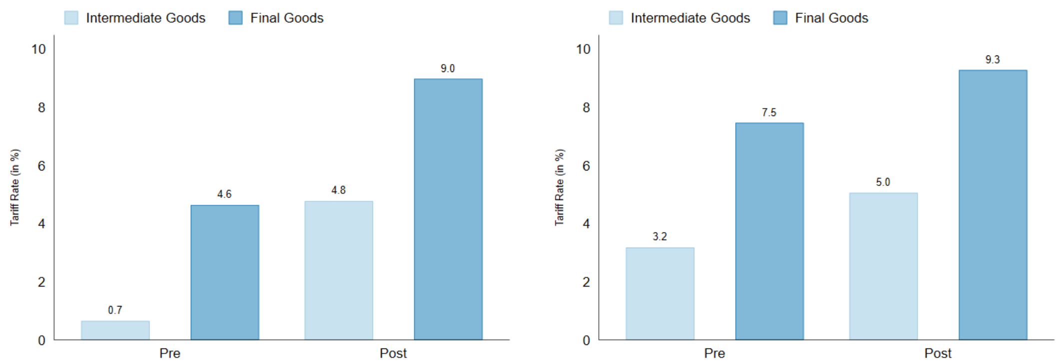
Figure 2 illustrates the general pattern of tariff escalation for both US tariffs on other countries and foreign tariffs on the United States, both before and after the recent trade war. Due to their low initial levels, average input tariffs increased the most on a percent basis, though tariff escalation remains as a distinct feature.[24]

Although we provide quantitative results for a set of parameters anchored on features of US

[24]Figure 2 displays the trade-weighted average tariffs on intermediate and final goods imposed by the United States on the Rest of the World (RoW) and by the RoW on US imports. Tariff data at the HTS 6-digit level for ROW are from the WTO Tariff Download Facility. Tariff data at the HTS 8-digit level for the United States are from the USITC. Import and export data are from the US Census Bureau. We classify goods into intermediate and final goods using the UN Broad Economic Categories (BEC). For details on data construction and for a version of Figure 2 without trade weights, see Online Appendix E.1.

Figure 2: Tariff Escalation Before and After the US-China Trade War

(a) US average tariffs on ROW **(b)** ROW average tariffs on US



Notes: Pre: Tariffs in January 2018, Post: Tariffs in December 2019. Tariff data from WTO and USITC, 2015 annual import and export data for weighted averages from US Census Bureau. Goods are classified as intermediate goods when their BEC code starts with 111, 121, 21, 22, 31, 322, 42 and 53. Final goods start with BEC code 41, 521, 112, 321, 522, 61, 62, 63 (including capital goods). All other goods have no classification.

data, the qualitative nature of our results – most notably, the fact that optimal tariffs on final goods are larger than on inputs – remains unaffected when exploring a wider range of parameter values as demonstrated in section 7.4. These results line up well with our theoretical results in sections 4 and 5, which is not entirely surprising given that the 'small open economy' assumption in those sections provides an accurate description of the US economy through the lens of our model.

7.1 Data and Parameters

In order to quantitatively discipline our model, we need to take a stance on a number of parameters of the model, and make sure that they provide values for key equilibrium variables consistent with those observed in the data. More specifically, the key parameters of the model are the elasticities of substitution upstream and downstream (θ and σ), the downstream labor share (α), iceberg trade costs upstream and downstream (τ^u and τ^d), productivity upstream and downstream in each country (A^u_{US}, A^u_{RoW}, A^d_{US} and A^d_{RoW}), fixed costs upstream and downstream (f^u and f^d), and the Home and Foreign labor endowments (L_{US} and L_{RoW}). Perhaps not too surprisingly given the isomoporphism we have developed in section 3.2, the fixed cost parameters turn out to be irrelevant for our quantitative conclusions, so we do not discuss them below.

Our quantitative approach constitutes a blend of calibration and estimation. We first discuss various approaches to estimating the key elasticities of substitution θ and σ, we then back out the downstream labor share α and the labor forces L_{US} and L_{RoW} from readily available public data, and we finally estimate trade costs (τ^u and τ^d) and the productivity parameters (A^u_{US}, A^u_{RoW}, A^d_{US} and A^d_{RoW}) by minimizing the distance between our model and a series of moments obtained from standard sources.

We next elaborate on the details of our approach.

Elasticities of Substitution (θ and σ) We consider four alternative approaches to quantifying the elasticities of substitution across varieties in the upstream and downstream sectors (θ and σ, respectively). We summarize these approaches here and provide additional details in Appendix E.2. The first approach is to treat these elasticities as symmetric across sectors. In this first approach, we fix the values of the elasticities of substitution across varieties in each sector to 5 ($\sigma = \theta = 5$), as in Costinot and Rodríguez-Clare (2014). We first consider this symmetric case to rule out the possibility that differences in demand elasticities across good types are the only source of variation in the response of welfare to changes in input versus final-good tariffs.

The second approach is to calibrate these parameters from data on mark-ups. Recall that under monopolistic competition and CES preferences the optimal firm-level mark-up is equal to $\theta/(\theta-1)$ upstream and $\sigma/(\sigma-1)$ downstream. Using sales and mark-up data from Baqaee and Farhi (2020) based on publicly listed firms in Compustat, we compute the sales-weighted average mark-ups of firms which we assigned to either upstream or downstream based on their primary sector. This approach leads to estimates of $\theta = 4.43$ for the elasticity of substitution upstream and of $\sigma = 6.44$ for the elasticity of substitution downstream.

The third approach is to estimate these parameters based on the response in trade flows to the US-China trade war in 2018-2019. Specifically, we follow Amiti et al. (2019) and calculate 12-month changes in US imports and US import tariffs at the product-country level. Exploiting the CES demand structure, regressing the changes in trade flows on the changes in tariffs provides estimates of the trade elasticity. Our preferred specification from this approach leads to estimates of $\theta = 2.35$ for the elasticity of substitution upstream and $\sigma = 3.08$ for the elasticity of substitution downstream. The small magnitude of the trade elasticities is consistent with the findings in Amiti et al. (2020) and could reflect that the response in trade flows was diminished by uncertainty about the persistence of these tariff changes.

The fourth (and final) approach is to exploit the isomorphism of our model to a competitive model with external economies of scale. As discussed in Section 2, these models are isomorphic provided that the following restrictions between the external economies of scale parameters and the elasticities of substitution across varieties hold: $\gamma^u = 1/(\theta-1)$ and $\gamma^d = 1/(\sigma-1)$. We use estimates of scale elasticities from Bartelme et al. (2019). We note two important caveats. First, they estimate these parameters only for 15 manufacturing sectors (we classify nine of these as upstream and six as downstream). Second, their framework abstracts from intermediate inputs and therefore their estimates may not be perfectly compatible with our setup. With these caveats in mind, the average (unweighted) scale elasticities are 0.133 upstream and 0.135 downstream. Exploiting the isomorphism between this setup and our framework with monopolistic competition and free entry, we convert these to $\theta = 8.52$ and $\sigma = 8.41$ for this fourth approach.

Downstream Labor Intensity, Trade and Expenditure Shares and Labor Endowments We measure the share of inputs in production, $1 - \alpha = 0.45$, from usage of intermediate inputs by downstream sectors based on the WIOD database (see Appendix E.3 for details). Similarly, we calculate trade and expenditure shares for the upstream (intermediate-input) and downstream (final consumption) sectors based on trade flow data provided in the WIOD, taking into account whether a trade flow is used for final consumption or as an intermediate input).[25] We infer the labor endowment of each country from population data published by CEPII.[26]

Estimation of Productivity Parameters and of Trade Costs Finally, we normalize US productivity in both sectors to one, $A^d_{US} = A^u_{US} = 1$. This leaves us with four parameters to estimate: trade costs in each sector $\{\tau^d, \tau^u\}$, and sectoral productivity in the rest of the world $\{A^d_{RoW}, A^u_{RoW}\}$.[27] To estimate the model, we search for the vector of parameters $\{\tau^d, \tau^u, A^d_{RoW}, A^u_{RoW}\}$ that minimizes the sum of squares of the differences between model-generated and empirical moments, subject to our equilibrium constraints. Panel B of Table 1 lists the set of moments we target in the estimation. The moments correspond to those that are necessary to solve for the changes in equilibrium outcomes in response to a counterfactual change in tariffs (i.e., the so-called hat algebra approach) and are all retrieved from the WIOD, as mentioned above.

Panel A in Table 1 presents the estimated values of the RoW's productivities and iceberg trade costs in each sector obtained under symmetric elasticities upstream and downstream, $\theta = \sigma = 5$. Trade costs appear slightly higher in the downstream sector, but within the range of standard estimates of trade barriers. The estimates indicate that the United States is about three times more efficient in final-good production than the rest of the world, and seven times more efficient in terms of input production. Despite only estimating four parameters, the fit of the model is quite good for most moments, except for the ratio of total sales in the upstream sector to total expenditure in the downstream sector. Note that in the data, the upstream sector uses intermediate inputs in production as well – which for simplicity we abstract from in our framework.

7.2 Decomposition of the Welfare Effects from Tariffs

To build intuition for our quantitative results, we begin by quantitatively evaluating the first-order welfare effects of input versus final-good tariffs following the approach developed in section 6. Using the calibrated model, we compute the decomposition of welfare changes in equation (38). To do so, we first solve for the zero-tariff equilibrium, so that we can compute the statistics Ω_{ij}, b^i_j and λ^d_i in this environment.[28]

Figure 3 depicts the welfare effects of changes in a final-good tariff (left panels) versus changes in an input tariff (right panels). The top two panels compare the percentage changes in welfare starting from the zero-tariff equilibrium (solid red line) to the percentage changes predicted by our

[25] We use data for 2014 which is the latest available year in the WIOD.

[26] Specifically, we set $L^{us} = 10 \times \frac{Pop^{us}}{Pop^{us}+Pop^{row}} = 0.45$ and $L^{row} = 10 \times \frac{Pop^{row}}{Pop^{us}+Pop^{row}} = 9.55$.

[27] We restrict entry costs f^d and f^u to be symmetric across sectors and countries and fix those values to 1. As anticipated before, this restriction is without loss of generality, as we have found both model fit and counterfactuals to be invariant when changing the entry costs to arbitrary (and possibly asymmetric) values.

[28] Under zero tariffs, these statistics take the values $\Omega_{H,H} = 0.41$, $\Omega_{F,H} = 0.04$, $\Omega_{F,F} = 0.44$, $\Omega_{H,F} = 0.02$, $b^H_H = 0.93$, $b^H_F = 0.07$, $\lambda^d_H = 0.98$.

Table 1: Calibrated Parameters and Moments

A. Calibrated Parameters	
Productivity in final-good sector, RoW relative to US, A^d_{row}	0.325
Productivity in input sector, RoW relative to US, A^u_{row}	0.142
Iceberg cost for final goods from US to RoW, τ^d	2.375
Iceberg cost for inputs from US to RoW, τ^u	2.032

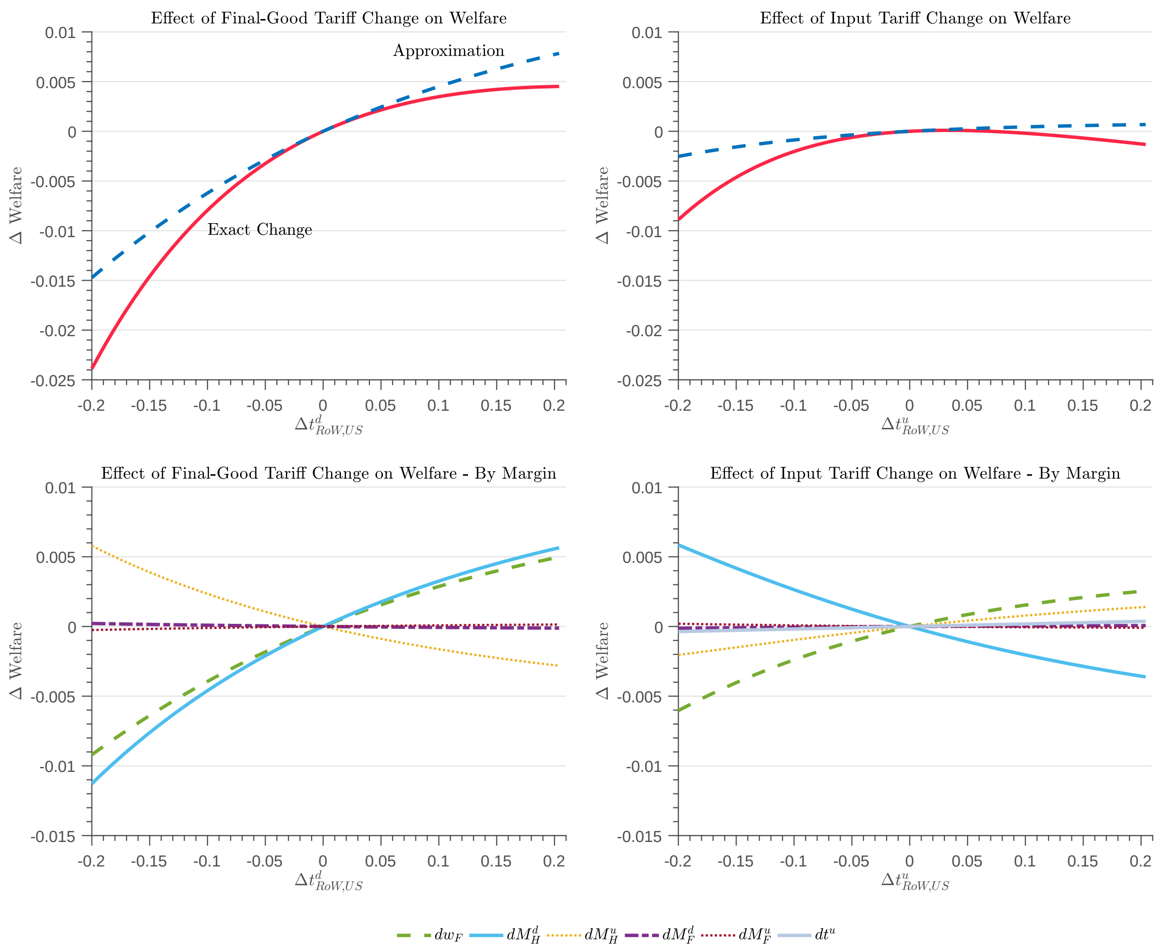
B. Moments	Data	Model
Sales share to US from US in final goods	0.943	0.964
Sales share to RoW from RoW in final goods	0.988	0.985
Sales share to US from US in intermediate good	0.897	0.889
Sales share to RoW from Row in intermediate good	0.982	0.978
Expenditure share in US final goods for the US	0.960	0.946
Expenditure share in RoW final good for the RoW	0.981	0.989
Expenditure share in US int. good for the US	0.906	0.921
Expenditure share in RoW int. good for the RoW	0.980	0.9670
Total US sales (int. goods) to total US expenditure (final goods)	0.771	0.466
Total RoW sales (int. goods) to total RoW expenditure (final goods)	1.242	0.446
Total US sales (final goods) to total US expenditure (final goods)	1.018	0.997
Total RoW sales (final goods) to total RoW expenditure (final goods)	0.993	0.999
Total expenditure in final goods by the US relative to RoW	0.303	0.285

Notes: Panel A presents the estimated values of the RoW's productivities and iceberg trade costs in each sector obtained under symmetric elasticities upstream and downstream, $\theta = \sigma = 5$. Panel B presents the targeted moments in the estimation. Column 1 presents moments from the data and column 2 presents their estimated counterparts. Note that in the model, total sales upstream to total expenditure downstream cannot be larger than 1 since the upstream sector is pure value added.

first-order approximations around zero (dashed-blue line). The first-order approximation works well for small changes in both final-good and intermediate-input tariffs. Starting from zero tariffs, the welfare effects of small import tariffs are positive for both types of goods, but turn negative for input tariffs at much lower rates than for final-good tariffs.

The bottom panels of Figure 3 decompose the approximation of the aggregate effects into their component parts, as shown in equation (38). Specifically, we decompose changes in welfare into changes due to: (i) changes in relative wages (dashed green); (ii) the relocation of final good producers to the United States (solid cyan); (iii) the relocation of input producers to the United States (dotted yellow); (iv) changes in the mass of final-good producers in the RoW (short-dash purple); (v) changes in the mass of input producers in the RoW (dash-dot magenta); and (vi) the gain from passing part of the input tariff onto final consumers in the RoW (solid gray). Although in the figure we label these by dw_F, dM^u_H, and so on, it should be understood that we are plotting the full value of each of the six terms in equation (38), with the labels identifying only one element

Figure 3: First Order Decomposition of Welfare Changes



Notes: Figure depicts the welfare effects of changes in a final-good tariff (left panel) versus changes in an input tariff (right panel). The top panels compare the percentage changes in utility starting from the zero-tariff equilibrium (solid red line) to the percentage changes predicted by our first-order approximations around zero (dashed blue line). The bottom two panels decompose the approximation of the aggregate effects into the component parts in equation (38). These are changes in welfare due to: (i) changes in relative wages (dashed green); (ii) the relocation of final good producers to the United States (solid cyan); (iii) the relocation of intermediate producers to the United States (dotted yellow); (iv) changes in the mass of final-good producers (short-dash purple); (v) changes in the mass of intermediate good producers (short-dash magenta); and (vi) the gain from passing part of the tariff onto foreign consumers (solid gray).

of each term.

Several observations are in order. First, notice that by raising tariffs on final goods, the US increases welfare not only because it tilts the factorial terms of trade in its favor – i.e., a reduction in w_F – but also because it induces a relocation of final-good producers into its own country. In addition, notice that the magnitude of the term associated with dM^d_H is on average as large as the term dw_F. Hence, this relocation effect is as important as the factorial terms-of-trade channel usually emphasized in the literature. Nevertheless, the net entry of final-good producers is accompanied by a net exit of input producers (yellow-dotted curve), with welfare effects that are about half as large as those associated with the relocation of final-good producers. The other three effects are largely negligible (the last one is exactly zero for the case of final-good tariffs). This confirms our previous claim that our quantitative results are consistent with the US being close to a small open economy, at least as defined in our model.

Turning to the results for input tariffs, we again see that the relocation effects are similar in magnitude to the factorial terms-of-trade effects, though these relocation effects now entail a net *negative* effect on welfare. Higher import tariffs on intermediate inputs increase entry upstream, but reduce entry of final-good producers. The negative welfare effect of the latter dominates quantitatively, which is intuitive based on a comparison of the terms multiplying dM_H^d and dM_H^u in equation (38), as explained in section 6. These terms contain the exposure of the consumer price index to changes in the measure of downstream and upstream producers, respectively.[29] The effect on welfare from changes in the mass of firms in the rest of the world is largely negligible, while the last term – the gain from passing intermediate-input tariffs on to foreign consumers – is small in magnitude.

7.3 Optimal Tariffs

In this section, we use the estimated parameters to compute the optimal tariff levels on final goods and intermediate inputs for the United States when the rest of the world sets a zero tariff on US goods.

Optimal Import Tariffs under First-Best Policies We begin by considering optimal policy in an environment in which the Home government has the necessary instruments to achieve the first-best allocation. We focus on the set of instruments discussed in Proposition 8, namely import tariffs and export taxes, as well as an upstream production subsidy. After normalizing the downstream export tax to zero (by Lerner's symmetry), we find that the optimal vector of policies is given by

$$\left(t_H^d, t_H^u, s_H^u, v_H^u\right) = (0.253, 0.003, 0.200, 0.200)\,.$$

In words, even when taking account general equilibrium effects coming from the US not being a small open economy, we find that the first-best policies are remarkably consistent with the results from Propostion 8. Tariff escalation is close to $\sigma/(\sigma-1)$, and the optimal domestic production subsidy upstream and the optimal upstream export tax are essentially indistinguishable from $1/\theta$. Note also that the upstream import tariff is virtually zero.

[29] Note that $\Omega_{H,H}$ and $\Omega_{H,F}$ are bounded above by $1-\alpha$. The welfare effect associated with a percentage increase in downstream firms is 0.31, whereas the term multiplying the percentage change in upstream firms is only 0.13 in equation (38).

Optimal Import Tariffs under Second-Best Policies Real world trade policies rarely feature export taxes—they are even disallowed by the US constitution—and production subsidies are rarely systematically used. For explaining the observed tariff escalation, second best policies that only feature import tariffs are therefore of particular interest. We again maximize US welfare, taking as given the the rest of the world place no tariffs on the US. We find that the vector of optimal import tariffs is given by

$$\left(t_H^d, t_H^u\right) = (0.306, 0.170) \, .$$

Tariff escalation thus prevails under second-best policies. Here, under $\alpha = 0.45$, tariff escalation is smaller in magnitude under second-best import tariffs compared to the first-best policy results above. Recall that in the $\alpha = 0$ case tariff escalation is larger under second best policies as shown in Proposition 4. Interestingly, however, optimal tariff escalation under second-best policies (around 1.11) is a bit larger in magnitude than the observed tariff escalation observed in the US in Figure 2 (around 1.04).

7.4 Optimal Tariffs: Robustness

We next briefly explore the robustness of our findings to alternative parameter values. We analyze optimal import tariffs under the three alternative procedures of estimating θ and σ as well as for different values for α (i.e., downstream value-added intensity). When changing these parameters, we re-calibrate the trade and productivity parameters to provide the best fit of the moments from Panel B of Table 1.

Table 2 provides extensive robustness tests of our optimal import tariffs in the case of first-best full policies when including a production subsidy and export tariffs as other instruments (panel A) and in the case of no other instruments except import tariffs (panel B). As is clear, the level of the tax instruments is quite sensitive to changes in the parameters. However, for all parameter values, we have that $\frac{1+t^d}{1+t^u} > 1$, and therefore optimal final-good tariffs are higher than input tariffs. Under the elasticity parameters shown in column 4, the optimal second-best tariff escalation is close in magnitude to the observed tariff escalation from Figure 2 (though the level of the import tariffs is much larger).

Panel A of Table 2 also reveal a systematic property of the tariff escalation consistent with the results from Proposition 8 derived for a small open economy. Comparing across columns, $\frac{1+t^d}{1+t^u} \approx \frac{\sigma}{\sigma-1}$. The pattern is striking, though note that this relationship is not exact, and can vary for a given level of σ as other parameters (e.g., α) are changed.

7.5 Counterfactuals: Evaluation of the 2018-2019 Tariff Increases

We close the paper by evaluating how the tariff increases during the 2018 to 2019 trade war – displayed in Figure 2 – affected US welfare. These tariff increases largely arose from the US-China trade war, but also include the US tariffs on washing machines, solar panels, aluminum, and steel, as

Table 2: Optimal Tax Policy - Robustness for Various Parameter Values

Parameter Values	$\theta = 4.43$ $\sigma = 6.44$	$\theta = 2.35$ $\sigma = 3.08$	$\theta = 8.52$ $\sigma = 8.41$	$\theta = 2.5$ $\sigma = 4$	$\theta = 5.5$ $\sigma = 4$	$\alpha = 0.75$	$\alpha = 0.25$	$\alpha = 0$
A. Optimal Trade & Tax Policies								
t^d	0.187	0.488	0.137	0.338	0.339	0.254	0.258	0.265
t^u	0.003	0.002	0.002	0.003	0.003	0.003	0.003	0.002
v^u	0.227	0.428	0.118	0.426	0.182	0.200	0.205	0.211
s^u	0.226	0.425	0.118	0.400	0.182	0.200	0.200	0
$\frac{1+t^d}{1+t^u}$	1.184	1.484	1.136	1.334	1.335	1.249	1.255	1.262
B. Optimal Import Tariffs								
t^d	0.224	0.505	0.162	0.365	0.388	0.260	0.334	0.349
t^u	0.176	0.314	0.091	0.301	0.151	0.182	0.111	0.056
$\frac{1+t^d}{1+t^u}$	1.042	1.145	1.065	1.049	1.205	1.065	1.201	1.278

Note: Each column presents optimal tariffs and taxes for alternative values of the parameters described in section 7.1 and their corresponding, re-estimated values of τ^d, τ^u, A^d_{row} and A^u_{row}. The set of calibrated parameters that corresponds to each column is displayed in Table F.1 in Appendix F.1. Panels A and B present optimal tariffs and taxes for the cases of policy instruments in Section 7.3. Tariff escalation ($\frac{1+t^d}{1+t^u} > 1$) is a robust feature across all specifications.

well as the retaliatory tariffs from the rest of the world (RoW). As our estimates for the elasticities of substitution upstream and downstream, we use the values obtained from the data on mark-ups resulting in $\theta = 4.43$ and $\sigma = 6.44$ (see Section 7.1).

Table 3 displays our results. We consider the case with and without tariff retaliation by the RoW, both of which are displayed in panels A and B, respectively. The US average import tariffs increased from 0.7 to 4.8 percentage points for intermediate goods, and from 4.6 to 9 percentage points for final goods. We find that without tariff retaliation by the RoW, US welfare would have increased by 0.12% from these tariff changes. The positive welfare gain for the US from the tariff increase is consistent with our estimates of the sizable unilateral optimal tariff rates for the US in Section 4 (in the absence of any export taxes or domestic subsidies).

We next evaluate the extent to which this gain was due to larger final-good versus input tariffs. In the third row of Table 3, we consider only the increase in the final good tariff, whereas row four considers the case if only the tariffs on intermediate goods had increased. The comparison reveals that the US welfare gains are driven overwhelmingly by higher final-good tariffs.

The dominant role of final-good tariffs on welfare increases is even more stark when considering a counterfactual increase in US final-good tariffs (row 5 of Table 3), that would have (naively) raised the same revenue as the observed tariff increases based on the average tariff rate change and the 2017 trade flows. In this case, US welfare – absent any foreign retaliation – would have risen by 0.11%. If instead the tariff increases had been adjusted so to apply only to intermediate inputs, US welfare would have increased by only 0.02%. Recall that all these figures are for a scenario without any foreign retaliation.

Table 3: Counterfactual Welfare Effects of US-China Trade War

	A. RoW tariff at 2017 level			**B.** RoW tariff at 2019 level		
	U_{US}	U_{RoW}	$\frac{U_{US}}{U_{US,2017}}$	U_{US}	U_{RoW}	$\frac{U_{US}}{U_{US,2017}}$
US tariff - 2017 level	0.057586	0.172282				
US tariff - 2019 level	0.057657	0.172125	1.001228	0.057598	0.172152	1.000217
2019 US tariff only Downstream	0.057632	0.172177	1.000791	0.057573	0.172206	0.999791
2019 US tariff only Upstream	0.057604	0.172235	1.000314	0.057548	0.172263	0.999343
Counterfactual Tariff only Downstream	0.057650	0.172097	1.001122	0.057592	0.172125	1.000106
Counterfactual Tariff only Upstream	0.057597	0.172145	1.000194	0.057541	0.172171	0.999233
Optimal US Import Tariffs	0.057733	0.171737	1.002544	0.057671	0.171757	1.001482
Optimal US Trade & Tax Policies	0.058374	0.171715	1.013684	0.058312	0.171736	1.012608

Notes: Table presents US welfare (U_{US}), RoW welfare (U_{RoW}), and US welfare in each counterfactual scenario relative to US welfare with actual 2017 tariffs ($\frac{U_{US}}{U_{US,2017}}$). Panel A computes welfare for various US tariff and tax policies holding the RoW tariffs at their 2017 levels. Panel B repeats the analysis, but using the observed 2019 RoW retaliatory tariffs. *US tariff - 2017 level* provides baseline welfare values using actual 2017 tariff values; *US tariff - 2019 level* uses actual 2019 US tariffs; *2019 US tariff only Downstream* uses 2017 upstream but 2019 downstream tariffs; *2019 US tariff only Upstream* uses 2017 downstream but 2019 upstream tariffs; *Counterfactual Tariff only Downstream (Upstream)* uses a counterfactual US downsteam (upstream) tariff that generates the same revenue as the actual 2019 US tariffs, based on the observed trade flows in 2017. Counterfactual tariffs are $\tilde{t}^d = 0.131$ or $\tilde{t}^u = 0.156$; *Optimal US Import Tariffs* uses the second-best optimal import tariffs in response to RoW's trade policy in 2017 (Panel A) or 2019 (Panel B). The optimal policy vector for panel A is $\left(t_H^d, t_H^u\right) = (0.231, 0.183)$ and $\left(t_H^d, t_H^u\right) = (0.232, 0.184)$ for panel B; *Optimal US Trade & Tax Policy*: US chooses optimal import tariffs, export tax, and production subsidy, as described in Section 4, in response to RoW's trade policy from 2017 (Panel A) or 2019 (Panel B). The optimal policy vector is $\left(t_H^d, t_H^u, v_H^u, s_H^u\right) = (0.186, .003, -0.227, 0.226)$ for panel A and $\left(t_H^d, t_H^u, v_H^u, s_H^u\right) = (0.186, 0.003, -0.227, 0.226)$ for panel B.

In Panel B of Table 3, we repeat these exercises but now take into consideration the observed retaliation by the RoW, which increased its import tariffs on US intermediate inputs from 3.2 to 5 percentage points and on US final goods from 7.5 to 9.3 percentage points. In this case, the US welfare gain from the tariff increases shrinks to only 0.02%. Therefore, tariff retaliation by the RoW largely undermines the US welfare gains from higher tariffs, which in turn are overwhelmingly driven by higher final-good tariffs (and not input tariffs). If the US had only placed the tariffs on final goods, while (naively) raising the same revenue (row 5), US welfare would have increased by 0.01%, even accounting for retaliation. If instead those tariffs had only been placed on intermediate inputs (row 6), US welfare would have declined by 0.06%.

Rows seven and eight present the potential welfare gains from implementing the optimal import tariffs with and without other tax policy instruments for the United States. Without allowing for production subsidies, the gains from optimal import tariffs are displayed in row seven of Table 3. Optimal US import tariffs absent any foreign retaliation could achieve a welfare gain of 0.25%.[30]

[30] The levels of these tariffs are much higher than the ones observed in the data $(t_d^*, t_u^*) = (0.2313, 0.1831)$.

Row eight allows for a full set of instruments that include both import and export taxes as well as production subsidies.[31] The optimal trade policy with domestic subsidies lead to a 1.4% increase in welfare in the case without retaliation by the RoW. We note that these figures assume no foreign retaliation (or, in panel B, no retaliation above the observed changes from RoW tariffs from 2017 to 2019).

8 Conclusion

In this paper, we provide an efficiency rationale for the fact that import tariffs on final good are systematically higher those on intermediate inputs. This so-called tariff escalation has been widely documented across time and space, but there is little support in the literature for the notion that this pattern constitutes a social welfare-maximizing policy.

We develop a two-sector model with a final-good sector and an intermediate input sector, both featuring increasing returns to scale, and show that (i) first-best trade policies are consistent with tariff escalation, and that (ii) second best import tariffs feature tariff escalation.

Although our model generically features domestic distortions related to the existence of scale economies upstream, the optimality of relatively lower input tariffs is *not* explained by a (second-best) correction of these domestic distortions. If anything, domestic distortions reduce the desirability of tariff escalation. Instead, input tariffs are less beneficial because they impact the size of the final-good sector *and* because the size of the final-good sector shapes its productivity under increasing returns to scale. It is thus scale economies downstream, rather than upstream, that shape the optimality of tariff escalation.

Our results are based on a parsimonious model featuring a single factor of production, only two sectors of production, and homogeneous firms. Future research should elucidate the robustness of our results to more realistic settings, which should also provide a helpful lens through which to interpret the drivers of tariff escalation in the data.

[31] The optimal policy has import taxes $(t_d^*, t_u^*) = (0.1860, 0.0027)$ and export taxes upstream, $v_u^* = 0.2269$ as well as production subsidies $s_u = 0.2261$.

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Trade Policy and Global Sourcing:
A Rationale for Tariff Escalation

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Online Appendix (Not for Publication)

A Closed-Economy Model: Details on Derivations

A.1 Equilibrium

Given the CES assumptions built into our framework and the lack of strategic interactions, firms in both sectors charge a constant markup over their marginal cost, which delivers

$$p^u = \frac{\theta}{\theta - 1} \frac{w}{A^u} \tag{A.1}$$

and

$$p^d = \frac{\sigma}{\sigma - 1} \frac{1}{A^d} \frac{w^\alpha (P^u)^{1-\alpha}}{\alpha^\alpha (1-\alpha)^{1-\alpha}}, \tag{A.2}$$

where P^u is the price index of intermediate inputs associated with Q^u, or

$$P^u = \left(\int_0^{M^u} p^u(\varpi)^{1-\theta} d\varpi \right)^{\frac{1}{1-\theta}}.$$

Firms make zero profits due to free entry, which pins down firm size according to:

$$x^u = (\theta - 1) f^u, \qquad x^d = (\sigma - 1) f^d. \tag{A.3}$$

Naturally, in equilibrium we must have $x^d = q^d$ and $x^u = M^d q^u$. The total measure of firms in the economy can be determined as follows. First, note that from the household's demand for downstream goods we have

$$M^d p^d q^d = wL, \tag{A.4}$$

which plugging in (A.2) and (A.3) delivers

$$M^d = \frac{\alpha^\alpha A^d}{f^d \sigma} \left((1-\alpha) \frac{\theta - 1}{\theta} A^u \right)^{1-\alpha} (M^u)^{\frac{1-\alpha}{\theta-1}} L. \tag{A.5}$$

Next, note that labor-market clearing imposes

$$L = M^u \frac{(f^u + x^u)}{A^u} + M^d \frac{\alpha p^d x^d}{w}. \tag{A.6}$$

Plugging in equations (A.3) and (A.4), we can solve for the total measure of firms in the upstream sector:

$$M^u = \frac{(1-\alpha) A^u L}{f^u \theta}. \tag{A.7}$$

Then, equations (A.5) and (6) determine the measure of firms in the downstream sector:

$$M^d = \frac{\alpha^\alpha A^d}{f^d \sigma} ((\theta - 1) f^u)^{1-\alpha} \left(\frac{(1-\alpha) A^u}{f^u \theta} \right)^{\frac{(1-\alpha)\theta}{\theta-1}} (L)^{\frac{\theta-\alpha}{\theta-1}}. \tag{A.8}$$

Finally, aggregate welfare is simply given by $U = \left(M^d\right)^{\frac{\sigma}{\sigma-1}} q^d$, where M^d is given in (7) and $q^d = x^d$ in (A.3).

When $\alpha \to 1$, we obtain

$$U = \left(\frac{A^d}{f^d \sigma} L\right)^{\frac{\sigma}{\sigma-1}} (\sigma - 1) f^d,$$

which is the standard formula in Krugman (1980).[1] Welfare is increasing in market size with an elasticity equal to $\frac{\sigma}{\sigma-1} > 1$, reflecting the variety gains associated with living in an economy that provides a larger number of final-good varieties.

Relative to this "Krugman" benchmark, in the presence of an active upstream sector (i.e., $\alpha < 1$), our model continues to feature scale effects, and in fact the elasticity of welfare to market size is larger than in the model with only a final-good sector. To see this, we can write welfare as

$$U = \left(\frac{(\sigma - 1)A^d/\sigma}{((\sigma - 1) f^d)^{\frac{1}{\sigma}}} \left(\frac{(\theta - 1)\, A^u/\theta}{((\sigma - 1) f^u)^{1/\theta}} \right)^{\frac{(1-\alpha)\theta}{\theta-1}} (L)^{\frac{\theta-\alpha}{\theta-1}} \right)^{\frac{\sigma}{\sigma-1}} \xi_\alpha, \tag{A.9}$$

where ξ_α is a function of only α and θ. Note that $\frac{\theta-\alpha}{\theta-1} \geq 1$, and thus this framework features larger scale effects than our model without an input sector.

To gain a better understanding of the role of imperfect competition and increasing returns to scale on welfare in our closed economy, we next turn to characterizing the social optimum in our model, and explore conditions under which the above market equilibrium is efficient.

A.2 Social Planner Problem

The social planner maximizes the objective in (1), choosing M^d, M^u, ℓ^d, ℓ^u, x^d and x^u subject to feasibility, or:

$$\begin{aligned} \max_{M^d, M^u, \ell^d, \ell^u, x^d, x^u} \quad & U = \left(M^d\right)^{\frac{\sigma}{\sigma-1}} x^d \\ s.t. \quad & L = \ell^u M^u + \ell^d M^d \\ & f^u + x^u = A^u \ell^u \\ & f^d + x^d = A^d \left(\ell^d\right)^\alpha \left((M^u)^{\frac{\theta}{\theta-1}} \frac{x^u}{M^d} \right)^{1-\alpha}. \end{aligned}$$

Working with the first-order conditions of this problem, we find that

$$(x^u)^* = (\theta - 1) f^u \tag{A.10}$$

and

$$(M^u)^* = \frac{\theta}{\theta - \alpha} \frac{(1-\alpha) A^u L}{\theta f^u} \tag{A.11}$$

in the upstream sector, and

$$\left(x^d\right)^* = (\sigma - 1) f^d \tag{A.12}$$

[1] A small and immaterial point of departure from Krugman (1980) is the fact that we have modeled the productivity terms A^d and A^u as shaping both the marginal and fixed costs of production. As a result, firm size is independent of these productivity parameters, but these parameters affect welfare directly.

and

$$\left(M^{d}\right)^{*}=\left(\frac{\theta-1}{\theta-\alpha}\right)^{\alpha}\left(\frac{\theta}{\theta-\alpha}\right)^{\frac{\theta(1-\alpha)}{\theta-1}}\frac{\alpha^{\alpha}A^{d}}{\sigma f^{d}}\left(\left(\theta-1\right)f^{u}\right)^{1-\alpha}\left(\frac{(1-\alpha)A^{u}}{\theta f^{u}}\right)^{\frac{(1-\alpha)\theta}{\theta-1}}(L)^{\frac{\theta-\alpha}{\theta-1}} \tag{A.13}$$

in the downstream sector. Comparing equations (A.10)-(A.13) to the corresponding ones in the market equilibrium, we conclude that:[2]

Proposition 1. In the decentralized equilibrium, firm-level output is at its socially optimal level in both sectors, but the market equilibrium features too little entry into both the downstream and upstream sectors unless $\alpha = 1$ (so the upstream sector is shut down) or $\alpha = 0$ (so the downstream sector does not use labor directly in production).

Why is the market equilibrium typically inefficient? At first glance, it may appear that the only source of inefficiency is the markup charged by upstream producers, which distorts the choice between labor and the bundle of input varieties for downstream firms. More specifically, this upstream markup makes inputs relatively more expensive and, in response, downstream firms substitute towards labor. At the same time, that markup also incentivizes entry upstream, which generates a variety-productivity effect downstream. To disentangle these two opposing forces, it is useful to compare the market allocation of labor to the social planner's optimal allocation.

Combining equations (2), (A.3), and (A.7), the aggregate decentralized market allocation of labor to the upstream sector is given by

$$M^{u}\ell^{u}=(1-\alpha)L,$$

while from equations (A.10) and (A.11), the social planner would allocate a share of labor to that sector equal to

$$M^{u}\ell^{u}=\frac{\theta}{\theta-\alpha}(1-\alpha)L>(1-\alpha)L.$$

Thus, the market equilibrium is inefficient, in the sense that it allocates too little labor to the upstream sector. It might seem intuitive that this inefficiency is associated with upstream markups leading to a double-marginalization inefficiency. However, note that the market allocation of labor to the upstream sector is actually *independent* of the degree of input substitutability (θ), and thus, of the level of upstream markups. In other words, lower input substitutability – and thus higher markups – do *not* depress the market allocation of labor to the upstream sector; instead, they increase the social-welfare maximizing allocation of labor to that sector. This fact does not necessarily rule out the relevance of a double marginalization inefficiency, but it does suggest that the market inefficiency may alternatively be interpreted as reflecting that, in the market equilibrium, upstream firms do not internalize the fact that their entry generates positive spillovers for firms in the downstream sector, with the size of this spillover decreasing in the degree of input substitutability θ.[3]

When $\alpha = 1$ or $\alpha = 0$, all labor is allocated to either the downstream sector (when $\alpha = 1$) or to the upstream sector (when $\alpha = 0$), and because firm-level output is always efficient, there is no scope for a market inefficiency in those two cases.

[2] Notice that for $\theta > 1$, $\left(\frac{\theta-1}{\theta-\alpha}\right)^{\alpha}\left(\frac{\theta}{\theta-\alpha}\right)^{\frac{\theta(1-\alpha)}{\theta-1}} \geq 1$, with equality when α is either 0 or 1.

[3] This can be verified from the fourth constraint of the social planner problem above, which indicates that downstream productivity is proportional to $(M^{u})^{\frac{\theta(1-\alpha)}{\theta-1}}$.

A.3 Optimal Policy

Suppose we endow a government with the ability to provide subsidies (or charge taxes) on the purchases of final goods or intermediate inputs. Denote these taxes by s^d and s^u in the downstream and upstream sectors, respectively. We assume that subsidy proceeds are extracted from households (or tax revenue is rebated to households) in a lump-sum manner.

Once we introduce taxes, price indexes become:

$$P^u = (M^u)^{\frac{1}{1-\theta}} (1 - s^u)p^u, \qquad P^d = \left(M^d\right)^{\frac{1}{1-\sigma}} (1 - s^d)p^d$$

and household disposable income becomes,

$$I = wL - M^d s^d p^d x^d - M^u s^u p^u x^u.$$

It is straightforward to show that taxes and subsidies do not alter the equilibrium firm size, which is still pinned down by free entry at the (efficient) levels given in (A.3). Turning to the determination of the measure of firms in each sector, we first invoke households' demand for downstream goods combined with goods-market clearing and household total income to obtain

$$M^d = \frac{wL - s^u M^u p^u x^u}{p^d x^d}.$$

Next, labor market clearing ensures that equation (A.6) still holds. The equilibrium measure of firms, given subsidies s^d and s^u, is then:

$$M^u = \frac{1}{1-\alpha s^u} \frac{(1-\alpha)A^u L}{\theta f^u}$$

$$M^d = (1 - s^u)^\alpha \left(\frac{1}{1-\alpha s^u}\right)^{\frac{\theta-\alpha}{\theta-1}} \frac{\alpha^\alpha A^d}{\sigma f^d} \left[\frac{(1-\alpha)A^u}{\theta f^u}\right]^{(1-\alpha)\frac{\theta}{\theta-1}} \left((\theta - 1) f^u\right)^{1-\alpha} L^{\frac{\theta-\alpha}{\theta-1}}.$$

Notice that downstream subsidies s^d have no impact on the market allocation. Because they are a *redundant* instrument, we can safely set them to 0. From the above expressions, it is then clear that:

Proposition 2. The social planner can restore efficiency in the market equilibrium by subsidizing upstream production at a rate $(s^u)^* = 1/\theta$.

Notice that the subsidy corresponds to the reciprocal of the elasticity of substitution across inputs. As a result, this subsidy encourages the entry of upstream suppliers especially when the inputs they produce are relatively less substitutable. There are two potential (and non-exclusive) explanations for this result. First, the lower is θ, the larger is the market power of and thus the markup charged by input suppliers, and thus the larger the subsidy required to undo this double marginalization inefficiency. Second, the lower is θ, the larger are the variety gains associated with upstream entry on the productivity of downstream firms, so to the extent that those gains are not internalized by input suppliers, again the larger is the required subsidy upstream.

A.4 Double Marginalization versus External Effects

We next dig a little bit deeper into the source of the market inefficiency. More specifically, we show that our vertical Krugman economy is isomorphic to a competitive vertical economy with external economies of scale. In this variant of our model, it is clear that the market inefficiency is due only to upstream suppliers failing to internalize the positive productivity effects of their entry on downstream firms (since there are no markups), and an upstream subsidy is again sufficient to restore efficiency.

The vertical economy with external economies of scale features consumers that spend their income on a single homogeneous final good. On the production side, this final good is produced combining labor and a homogeneous intermediate input, which is in turn produced with labor. The homogeneous intermediate input and final good are produced according to the technologies

$$
\begin{aligned}
x^u &= A^u \ell^u \left(L^u\right)^{\gamma^u} \\
x^d &= A^d \left(\ell^d\right)^{\alpha} \left(q^u\right)^{1-\alpha} \left(\left(L^d\right)^{\alpha} \left(Q^u\right)^{1-\alpha}\right)^{\gamma^d},
\end{aligned}
$$

where L^u and L^d are the aggregate allocations of labor to the upstream and downstream sector, Q^u is total production upstream, and γ^u and γ^d measure the magnitude of external economies of scale.

Individual firms are symmetric, competitive, and infinitessimal, so they take the aggregates as given and price at marginal cost. The resulting prices for the upstream and downstream sector are given by

$$P^u = \frac{w}{A^u} \left(L^u\right)^{-\gamma^u}$$

and

$$P^d = \frac{1}{A^d} \left(\frac{w}{\alpha}\right)^{\alpha} \left(\frac{P^u}{1-\alpha}\right)^{1-\alpha} \left(\left(L^d\right)^{\alpha} \left(Q^u\right)^{1-\alpha}\right)^{-\gamma^d}.$$

Invoking $P^d Q^d = wL$, $Q^u = A^u \left(L^u\right)^{1+\gamma^u}$ and $Q^d = A^d \left(\left(L^d\right)^{\alpha} \left(Q^u\right)^{1-\alpha}\right)^{1+\gamma^d}$, it is straightforward to infer that the equilibrium allocation of labor across sectors is given by

$$L^u = (1-\alpha) L, \quad \text{and} \quad L^d = \alpha L,$$

just as in our "Krugman" vertical economy with internal economies of scale. In addition, one can also show that whenever $\gamma^u = 1/(\theta - 1)$ and $\gamma^d = 1/(\sigma - 1)$, with an appropriate choice of the technological parameters A^d and A^u, the equilibria of the two models are fully isomorphic, not just in terms of the allocation of labor across sectors, but also in terms of prices and welfare.

As in the case of internal economies of scale, this market equilibrium can easily be shown to be inefficient. In particular, setting up the planner problem,

$$
\begin{aligned}
\max_{L^u, L^d} \quad & Q^d = A^d \left(\left(L^d\right)^{\alpha} \left(Q^u\right)^{1-\alpha}\right)^{1+\gamma^d} \\
s.t. \quad & Q^u = A^u \left(L^u\right)^{1+\gamma^u} \\
& L^u + L^d = L,
\end{aligned}
\tag{A.14}
$$

it is straightforward to see that this delivers

$$\left(L^u\right)^* = \frac{1+\gamma^u}{\gamma^u (1-\alpha) + 1} (1-\alpha) L, \quad \text{and} \quad \left(L^d\right)^* = \frac{1}{\gamma^u (1-\alpha) + 1} \alpha L. \tag{A.15}$$

Clearly, the market equilibrium features too little labor allocated to the upstream sector whenever $\gamma^u > 0$ and $0 < \alpha < 1$. Finally, one can also verify that an upstream subsidy equal to $\left(s^u\right)^* = \gamma^u / (1 + \gamma^u)$ is sufficient to restore efficiency. While it is perhaps surprising that the planner need not make any correction for the external economies in the downstream sector, this result is due to the fact that all of that sector's output is sold to consumers. By contrast, the fact that inputs are all sold to firms means that their underprovision requires a subsidy so that it does not distort final-good producers' purchases of inputs versus labor.

In sum, we have shown that a model with external economies of scales is isomorphic to our model with internal economies of scale as long as $\gamma^u = 1/(\theta - 1)$, and that the rationale for the use of upstream subsidies to restore efficiency can be tied to a love-for-variety productivity effect, rather than it being necessarily driven by a double marginalization inefficiency.

A.5 Extensions

In this Appendix, we briefly develop two extensions of our closed-economy model, both featuring a more complex input sector.

I. Roundabout Production Upstream

We first allow the upstream sector to use not only labor in production, but also the same bundle of inputs Q^u used in the final-good sector. More specifically, and focusing on the isomorphic economy with perfect competition, homogeneous goods, and external economies of scale developed in section A.4, we now assume

$$\begin{aligned} x^u &= A^u \left(\ell^u\right)^{\beta} \left(q^u\right)^{1-\beta} \left(\left(L^u\right)^{\beta} \left(Q^u\right)^{1-\beta}\right)^{\gamma^u} \\ x^d &= A^d \left(\ell^d\right)^{\alpha} \left(q^u\right)^{1-\alpha} \left(\left(L^d\right)^{\alpha} \left(Q^u\right)^{1-\alpha}\right)^{\gamma^d}, \end{aligned}$$

where β governs the labor instensity of production upstream. It is clear from the second of these expressions that firms in the downstream sector will spend a fraction of its costs on the upstream sector, or

$$P^u q^u = (1-\alpha) P^d x^d.$$

Noting that, due to symmetry, $x^u = Q^u$ and $x^d = Q^d$, and that the decentralized market prices for the downstream sector is given by

$$P^d = \frac{1}{A^d} \left(\frac{w}{\alpha}\right)^{\alpha} \left(\frac{P^u}{1-\alpha}\right)^{1-\alpha} \left(\left(L^d\right)^{\alpha} \left(Q^u\right)^{1-\alpha}\right)^{-\gamma^d}.$$

Invoking $P^d Q^d = wL$ and $Q^d = A^d \left(\left(L^d\right)^{\alpha} \left(Q^u\right)^{1-\alpha}\right)^{1+\gamma^d}$ we thus obtain

$$\frac{1}{A^d} \left(\frac{w}{\alpha}\right)^{\alpha} \left(\frac{P^u Q^u}{1-\alpha}\right)^{1-\alpha} A^d \left(L^d\right)^{\alpha} = wL$$

or

$$\left(\frac{w}{\alpha}\right)^{\alpha} (wL)^{1-\alpha} \left(L^d\right)^{\alpha} = wL,$$

from which it is immediate that

$$L^u = (1-\alpha) L, \quad \text{and} \quad L^d = \alpha L,$$

just as in our baseline model

We next consider the planner problem,

$$\max_{L^u, L^d} \quad Q^d = A^d \left(\left(L^d\right)^{\alpha} \left(Q^u\right)^{1-\alpha} \right)^{1+\gamma^d}$$
$$s.t. \quad Q^u = A^u \left(\left(L^u\right)^{\beta} \left(Q^u\right)^{1-\beta} \right)^{1+\gamma^u}$$
$$L^u + L^d = L.$$

Noting that the second constraint can be written as

$$Q^u = \tilde{A}^u \left(L^u\right)^{1+\tilde{\gamma}^u},$$

where

$$\begin{aligned} \tilde{A}^u &= \left(A^u\right)^{\frac{1}{1-(1-\beta)(1+\gamma^u)}} \\ \tilde{\gamma}^u &= \frac{\gamma^u}{1-(1-\beta)\left(1+\gamma^u\right)}, \end{aligned}$$

it then becomes clear that this program is identical to the one in our baseline model, except for the fact that the scale elasticity upstream is now not given by γ^u, but by $\tilde{\gamma}^u > \gamma^u$ (the program also features a rescaled upstream productivity, but that is immaterial). Note that the gap between $\tilde{\gamma}^u$ and γ^u is decreasing in β.

Analogously to equation (A.15), the socially optimal allocation of labor is given by

$$\left(L^u\right)^* = \frac{1+\tilde{\gamma}^u}{\tilde{\gamma}^u\left(1-\alpha\right)+1}\left(1-\alpha\right)L, \quad \text{and} \quad \left(L^d\right)^* = \frac{1}{\tilde{\gamma}^u\left(1-\alpha\right)+1}\alpha L.$$

Clearly, the market equilibrium features too little labor allocated to the upstream sector whenever $\gamma^u > 0$ and $0 < \alpha < 1$, just as in our baseline model, but the inefficiency is now decreasing in β. Finally, one can also verify that an upstream subsidy equal to $\left(s^u\right)^* = \tilde{\gamma}^u / \left(1+\tilde{\gamma}^u\right)$ is sufficient to restore efficiency. Because $\tilde{\gamma}^u > \gamma^u$, this subsidy is now higher than in our baseline model, and it is decreasing in β.

II. Multi-Stage Production

We next develop a multi-stage extension of the model. We begin with a simple three-stage production process with a downstream sector, a midstream sector, and an upstream sector. The technologies are given by

$$\begin{aligned} x^u &= A^u \left(\ell^u\right) \left(L^u\right)^{\gamma^u} \\ x^m &= A^d \left(\ell^m\right)^{\beta} \left(q^u\right)^{1-\beta} \left(\left(L^m\right)^{\beta} \left(Q^u\right)^{1-\beta} \right)^{\gamma^m} \\ x^d &= A^d \left(\ell^d\right)^{\alpha} \left(q^m\right)^{1-\alpha} \left(\left(L^d\right)^{\alpha} \left(Q^m\right)^{1-\alpha} \right)^{\gamma^d}, \end{aligned}$$

Using the fact that, in a decentralized equilibrium, we have

$$\begin{aligned} P^d Q^d &= wL; \\ Q^d &= A^d \left(\left(L^d\right)^\alpha (Q^m)^{1-\alpha} \right)^{1+\gamma^d}; \\ P^d &= \frac{1}{A^d} \left(\frac{w}{\alpha}\right)^\alpha \left(\frac{P^m}{1-\alpha}\right)^{1-\alpha} \left(\left(L^d\right)^\alpha (Q^m)^{1-\alpha} \right)^{-\gamma^d}; \\ P^m Q^m &= (1-\alpha) P^d Q^d, \end{aligned}$$

we immediately obtain

$$L^d = \alpha L.$$

Next, because

$$\begin{aligned} P^m Q^m &= (1-\alpha) wL; \\ Q^m &= A^m \left((L^m)^\alpha (Q^u)^{1-\alpha} \right)^{1+\gamma^m}; \\ P^m &= \frac{1}{A^m} \left(\frac{w}{\beta}\right)^\beta \left(\frac{P^u}{1-\beta}\right)^{1-\beta} \left((L^m)^\beta (Q^u)^{1-\beta} \right)^{-\gamma^m}; \\ P^u Q^u &= (1-\beta) P^m Q^m, \end{aligned}$$

we obtain

$$L^m = \beta (1-\alpha) L, \quad \text{and} \quad L^u = (1-\beta)(1-\alpha) L.$$

Now consider the planner problem

$$\begin{aligned} \max_{L^u, L^m, L^d} \quad & Q^d = A^d \left(\left(L^d\right)^\alpha (Q^m)^{1-\alpha} \right)^{1+\gamma^d} \\ s.t. \quad & Q^m = A^m \left((L^m)^\beta (Q^u)^{1-\beta} \right)^{1+\gamma^u} \\ & Q^u = A^u (L^u)^{1+\gamma^u} \\ & L^u + L^m + L^d = L. \end{aligned}$$

which delivers

$$\begin{aligned} (L^u)^* &= \frac{(1+\gamma^u)(1+\gamma^m)}{\alpha + (1-\alpha)(1+\gamma^m)(\beta + (1-\beta)(1+\gamma^u))} (1-\beta)(1-\alpha) L \\ (L^m)^* &= \frac{1+\gamma^m}{\alpha + (1-\alpha)(1+\gamma^m)(\beta + (1-\beta)(1+\gamma^u))} \beta (1-\alpha) L \\ \left(L^d\right)^* &= \frac{1}{\alpha + (1-\alpha)(1+\gamma^m)(\beta + (1-\beta)(1+\gamma^u))} \alpha L. \end{aligned}$$

Notice that the gap between the socially optimal and the market allocation of labor is higher the more upstream the stage. Does that mean that subsidies are higher, the more upstream a sector?

Consider the following key conditions identify a market equilibrium with subsidies:

$$\begin{aligned}
P^dQ^d &= wL - s^m P^m Q^m - s^u P^u Q^u \\
Q^d &= A^d \left(\left(L^d\right)^{\alpha} \left(Q^m\right)^{1-\alpha}\right)^{1+\gamma^d}; \\
P^d &= \frac{1}{A^d}\left(\frac{w}{\alpha}\right)^{\alpha} \left(\frac{\left(1-s^m\right)P^m}{1-\alpha}\right)^{1-\alpha} \left(\left(L^d\right)^{\alpha} \left(Q^m\right)^{1-\alpha}\right)^{-\gamma^d}; \\
\left(1-s^m\right)P^mQ^m &= \left(1-\alpha\right)P^dQ^d \\
Q^m &= A^m \left(\left(L^m\right)^{\alpha} \left(Q^u\right)^{1-\alpha}\right)^{1+\gamma^m} \\
P^m &= \frac{1}{A^m}\left(\frac{w}{\beta}\right)^{\beta} \left(\frac{\left(1-s^u\right)P^u}{1-\beta}\right)^{1-\beta} \left(\left(L^m\right)^{\beta} \left(Q^u\right)^{1-\beta}\right)^{-\gamma^m} \\
\left(1-s^u\right)P^uQ^u &= \left(1-\beta\right)P^mQ^m
\end{aligned}$$

Note that

$$P^dQ^d = wL - \frac{s^m}{1-s^m}\left(1-\alpha\right)P^dQ^d - \frac{s^u}{1-s^u}\frac{\left(1-\beta\right)}{1-s^m}\left(1-\alpha\right)P^dQ^d$$

or

$$P^dQ^d = \frac{wL}{1+\frac{s^m}{1-s^m}\left(1-\alpha\right)+\frac{s^u}{1-s^u}\frac{(1-\beta)}{1-s^m}\left(1-\alpha\right)}.$$

Next

$$\begin{aligned}
P^dQ^d &= \left(\frac{wL^d}{\alpha}\right)^{\alpha}\left(\frac{\left(1-s^m\right)P^mQ^m}{1-\alpha}\right)^{1-\alpha} \\
&= \left(\frac{wL^d}{\alpha}\right)^{\alpha}\left(P^dQ^d\right)^{1-\alpha},
\end{aligned}$$

so

$$\frac{L^d}{L} = \frac{\alpha}{1+\frac{s^m}{1-s^m}\left(1-\alpha\right)+\frac{s^u}{1-s^u}\frac{(1-\beta)}{1-s^m}\left(1-\alpha\right)}.$$

Next,

$$\begin{aligned}
P^mQ^m &= \left(\frac{wL^m}{\beta}\right)^{\beta}\left(\frac{\left(1-s^u\right)P^uQ^u}{1-\beta}\right)^{1-\beta} \\
&= \left(\frac{wL^m}{\beta}\right)^{\beta}\left(P^mQ^m\right)^{1-\beta},
\end{aligned}$$

so

$$P^mQ^m = \frac{\left(1-\alpha\right)}{\left(1-s^m\right)}P^dQ^d = \frac{wL^m}{\beta}$$

or

$$\frac{L^m}{L} = \frac{\beta\frac{(1-\alpha)}{(1-s^m)}}{1+\frac{s^m}{1-s^m}\left(1-\alpha\right)+\frac{s^u}{1-s^u}\frac{(1-\beta)}{1-s^m}\left(1-\alpha\right)}.$$

We thus have that the subsidies s^m and s^u need to satisfy

$$\frac{\alpha}{1+\frac{s^m}{1-s^m}\left(1-\alpha\right)+\frac{s^u}{1-s^u}\frac{(1-\beta)}{1-s^m}\left(1-\alpha\right)} = \frac{1}{\alpha+\left(1-\alpha\right)\left(1+\gamma^m\right)\left(\beta+\left(1-\beta\right)\left(1+\gamma^u\right)\right)}\alpha$$

and

$$\frac{\beta \frac{(1-\alpha)}{(1-s^m)}}{1+\frac{s^m}{1-s^m}(1-\alpha)+\frac{s^u}{1-s^u}\frac{(1-\beta)}{1-s^m}(1-\alpha)}=\frac{1+\gamma^m}{\alpha+(1-\alpha)(1+\gamma^m)(\beta+(1-\beta)(1+\gamma^u))}\beta(1-\alpha),$$

which delivers

$$s^m=\frac{\gamma^m}{1+\gamma^m};\quad s^u=\frac{\gamma^u}{1+\gamma^u}.$$

As is clear from this expression, subsidies in all sectors producing inputs are positive, but notice that subsidies are higher upstream relative to midstream only if $\gamma^u > \gamma^m$, i.e., only if the scale elasticity is higher upstream than midstream. This contrasts with the results of Liu (2019), who finds that optimal subsidies should necessarily be higher, the more upstream the sector. The reason is that, unlike in Liu's work, we solve for first-best subsidy policy: when the government can only set subsidies in one sector, the size of the subsidy would be higher, the more upstream the sector, because as we have seen above, the gap between the social optimal and market allocation of labor is highest in the upstream sector.

B Open Economy Model: Details on Derivations

B.1 Open Economy Equilibrium with Internal Economies of Scale

In this Appendix, we outline the equilibrium conditions corresponding to the two-country model featuring internal scale economies, product differentiation and monopolistic competition outlined in section 3.1. We will then work with these equations in Appendix to demonstrate the isomorphism claimed in Proposition 3.

Import tariffs on the downstream sector create a wedge between consumer prices in country i and producer prices in country j. More specifically, given CES preferences, consumer prices in i for goods originating in j are given by:

$$p_{ji}^{d} = \left(1+t_{i}^{d}\right) \frac{\sigma}{\sigma-1} \tau^{d} \frac{1}{A_{j}^{d}} \left(\frac{w_{j}}{\alpha}\right)^{\alpha} \left(\frac{P_{j}^{u}}{1-\alpha}\right)^{1-\alpha} = \frac{1+t_{i}^{d}}{1-v_{j}^{d}} \tilde{p}_{ji}^{d}. \tag{B.1}$$

Similarly, import tariffs on the upstream sector create a wedge between the price paid by final-good producers in i for inputs from j, and the producer price for those inputs obtained by suppliers in country j. In particular, we have

$$p_{ji}^{u} = \left(1+t_{i}^{u}\right) \frac{\theta}{\theta-1} \tau^{u} \frac{w_{j}}{A_{j}^{u}} = \frac{1+t_{i}^{u}}{1-v_{j}^{u}} \tilde{p}_{ji}^{u}. \tag{B.2}$$

In equation (B.1), the price index P_{i}^{u} is given by

$$P_{i}^{u} = \left[\sum_{j\in\{H,F\}} \left(P_{ji}^{u}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}, \tag{B.3}$$

with

$$P_{ji}^{u} = \left[\int_{0}^{M_{j}^{u}} \left(p_{ji}^{u}\left(\varpi\right)\right)^{1-\theta} d\varpi\right]^{\frac{1}{1-\theta}}. \tag{B.4}$$

When setting $j=i$, the above pricing equations also characterize domestic prices in country j after setting $t_{i}^{d}=t_{i}^{u}=v_{i}^{d}=v_{i}^{u}=0$ and $\tau^{d}=\tau^{u}=1$. Note that $p_{ii}^{d}=\tilde{p}_{ii}^{d}$ and $p_{ii}^{u}=\tilde{p}_{ii}^{u}$.

Next, utility maximization implies that when consuming country j varieties, consumers in i allocate to each variety ω a share of spending equal to

$$\frac{p_{ji}^{d}\left(\omega\right) q_{ji}^{d}\left(\omega\right)}{P_{ji}^{d} Q_{ji}^{d}} = \left(\frac{p_{ji}^{d}\left(\omega\right)}{P_{ji}^{d}}\right)^{1-\sigma}, \tag{B.5}$$

of their total spending on country j varieties, where

$$P_{ji}^{d} = \left[\int_{0}^{M_{j}^{d}} \left(p_{ji}^{d}\left(\omega\right)\right)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}. \tag{B.6}$$

Consumers' (aggregate) spending on Home and Foreign varieties is in turn determined by

$$P_{ji}^{d} Q_{ji}^{d} = \left(\frac{P_{ji}^{d}}{P_{i}^{d}}\right)^{1-\sigma} \left(w_{i} L_{i} + R_{i}\right), \tag{B.7}$$

where P_i^d is the aggregate consumer price index in i

$$P_i^d = \left[\sum_{j \in \{H,F\}} \left(P_{ji}^d \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \tag{B.8}$$

and R_i is tariff revenue, which we have defined in equation (10).

We now turn to profit maximization by downstream producers in country i. First note that free entry implies that firm revenue (net of tariffs) will equal total costs, and that a share of those costs will go to pay labor. As a result, labor compensation by each final-good producer in i is given by

$$w_i \ell_i^d = \alpha \left(\tilde{p}_{ii}^d q_{ii}^d + \frac{1 - v_i^d}{1 + t_j^d} \tilde{p}_{ij}^d q_{ij}^d \right). \tag{B.9}$$

Next, when purchasing inputs from upstream producers in country j, final-good producers in country i, will demand an amount of each variety ϖ from country j equal to

$$q_{ji}^u(\varpi) = Q_{ji}^u(\omega) \left(\frac{p_{ji}^u}{P_{ji}^u} \right)^{-\theta},$$

while aggregate spending on all country j's input varieties is given by

$$P_{ji}^u Q_{ji}^u = (1-\alpha) \left(\tilde{p}_{ii}^d q_{ii}^d + \frac{1 - v_i^d}{1 + t_j^d} \tilde{p}_{ij}^d q_{ij}^d \right) \left(\frac{P_{ji}^u}{P_i^u} \right)^{1-\theta} M_i^d. \tag{B.10}$$

Aggregate spending on Home and Foreign intermediate inputs in country i is then given by

$$P_i^u Q_i^u = (1-\alpha) \left(\tilde{p}_{ii}^d q_{ii}^d + \frac{1 - v_i^d}{1 + t_j^d} \tilde{p}_{ij}^d q_{ij}^d \right) M_i^d. \tag{B.11}$$

Our final set of equilibrium conditions impose market clearing. First, labor-market clearing in both countries implies that

$$L_i = M_i^d \ell_i^d + M_i^u \ell_i^u, \tag{B.12}$$

where ℓ_i^d is given in (B.9), and $\ell_i^u = (f_i^u + x_i^u)/A_i^u$.[4] Second, goods-market clearing imposes

$$q_{ii}^d + \tau^d q_{ij}^d = x_i^d \tag{B.13}$$

and

$$M_i^d q_{ii}^u + M_j^d \tau^u q_{ij}^u = x_i^u. \tag{B.14}$$

Note that free entry upstream and downstream implies that firm revenue is equal to total costs, which delivers

$$x_i^d = (\sigma - 1) f_i^d; \qquad x_i^u = (\theta - 1) f_i^u \tag{B.15}$$

for $i = \{H, F\}$. Firm-level production levels are thus independent of tariff choices, and the only way in which tariffs can affect the allocation of labor across sectors is by changing the measure of firms in each of the two

[4]Naturally, equilibrium also requires trade balance, but this is ensured by the other equilibrium conditions outlined in this section.

sectors. As a result, optimal trade policies will seek to achieve a social-welfare maximizing allocation of labor across sectors, with no concern for the allocation of labor within sectors (across fixed costs of entry versus marginal costs of production).

Despite the simple structure of the model and relatively simple equilibrium conditions, an analysis of how the market equilibrium is affected by input and final-good tariffs set by the Home country is complex, so we begin by considering the special case in which downstream production only uses inputs (and no labor) in production, or $\alpha = 0$.

B.2 Equilibrium of Isomorphic Competitive Economy with External Economies of Scale

In this Appendix we prove the isomorphism claimed in Proposition 3. More specifically, our goal is to show that equilibrium conditions of the decentralized equilibrium of the two-country model in section 3.1 featuring internal scale economies, product differentiation and monopolistic competition can be reduced to a set of equations identical to equations (14) through (21) applying to the competitive model with external economies of scale developed in this section.

Preferences We begin by noting that given symmetry in final-good production, we can express preferences as

$$\begin{aligned} U_i &= \left[\sum_{j \in \{H,F\}} \left(\int_0^{M_j^d} q_{ji}^d(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right) \right]^{\frac{\sigma}{\sigma-1}} \\ &= \left[M_i^d (q_{ii}^d)^{\frac{\sigma-1}{\sigma}} + M_j^d (q_{ji}^d)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \\ &= \left(\left(Q_{ii}^d\right)^{\frac{\sigma-1}{\sigma}} + \left(Q_{ji}^d\right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \end{aligned}$$

where

$$Q_{ii}^d \equiv \left(M_i^d\right)^{\frac{\sigma}{\sigma-1}} q_{ii}^d; \quad Q_{ji}^d \equiv \left(M_j^d\right)^{\frac{\sigma}{\sigma-1}} q_{ji}^d. \tag{B.16}$$

Starting from (9), we have thus derived (11), which are preferences in the isomorphic economy with two final goods (a Home one and a Foreign one) and external economies of scale.

Labor-Market Clearing Next, remember that ℓ_i^d and ℓ_i^u are the firm-level amounts of labor used downstream and upstream to cover fixed and variable costs. Hence, defining

$$L_i^d \equiv M_i^d \ell_i^d; \quad L_i^u \equiv M_i^u \ell_i^u, \tag{B.17}$$

we have that equation (B.12) in the monopolistic competition model implies equation (14) in the external economies model, or

$$L_i = M_i^d \ell_i^d + M_i^u \ell_i^u = L_i^u + L_i^d.$$

Upstream Market Clearing and Upstream Endogenous Productivity Next let us define

$$Q_{ii}^u \equiv M_i^d \left(M_i^u\right)^{\frac{\theta}{\theta-1}} q_{ii}^u; \quad Q_{ij}^u \equiv M_j^d \left(M_i^u\right)^{\frac{\theta}{\theta-1}} q_{ij}^u. \tag{B.18}$$

Given these definitions in (B.18), and given the definition of the input aggregate $Q_i^u(\omega)$ in the monopolistic competition model, that is

$$Q_i^u(\omega) = \left[\sum_{j\in\{H,F\}} \left(\int_0^{M_j^u} q_{ji}^u(\varpi)^{\frac{\theta-1}{\theta}} d\varpi \right) \right]^{\frac{\theta}{\theta-1}}, \qquad \theta > 1, \quad i \in \{H, F\},$$

we have that the total usage of inputs by firms in country i is given by

$$\begin{aligned} Q_i^u &= M_i^d Q_i^u(\omega) = \left[M_i^u \left(q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + M_j^u \left(q_{ji}^u\right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \\ &= \left[\left(M_i^d \left(M_i^u\right)^{\frac{\theta}{\theta-1}} q_{ii}^u \right)^{\frac{\theta-1}{\theta}} + \left(M_i^d \left(M_j^u\right)^{\frac{\theta}{\theta-1}} \left(q_{ji}^u\right) \right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \\ &= \left[\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \end{aligned} \tag{B.19}$$

and thus is analogous to a CES aggregator of only two inputs: a Home and a Foreign one, as defined in equation (B.18). These inputs are either produced domestically or are imported.

Now consider the domestic production of those inputs. Let us start from the definition of upstream technology in the monopolistic competition model, that is

$$f_i^u + x_i^u(\varpi) = A_i^u \ell_i^u(\varpi), \qquad \varpi \in [0, M_i^u], \quad i \in \{H, F\}.$$

Imposing symmetry and firm-level output in equation (B.15) – i.e., $x_i^u = (\theta - 1) f_i^u$ –, and invoking the definition of L_i^u in (B.17), we have

$$X_i^u \equiv \left(M_i^u\right)^{\frac{\theta}{\theta-1}} x_i^u = \left(\frac{A_i^u}{\theta f_i^u} \right)^{\frac{\theta}{\theta-1}} (\theta - 1) f_i^u \left(L_i^u\right)^{\frac{\theta}{\theta-1}} \tag{B.20}$$

or

$$X_i^u = \hat{A}_i^u F_i^u \left(\ell_i^u\right) = \bar{A}_i^u \left(L_i^u\right)^{1+\gamma^u},$$

where

$$\bar{A}_i^u \equiv \left(\frac{A_i^u}{\theta f_i^u} \right)^{\frac{\theta}{\theta-1}} (\theta - 1) f_i^u,$$

and

$$\gamma^u \equiv 1/\left(\theta - 1\right).$$

Because this domestic production X_i^u is sold domestically or exported, we have

$$\bar{A}_i^u \left(L_i^u\right)^{1+\gamma^u} = Q_{ii}^u + Q_{ij}^u,$$

which corresponds exactly to equation (15) in the external economies model.

Downstream Market Clearing and Downstream Endogenous Productivity We can proceed analogously for final-good production. We begin with the definition of technology in the downstream sector in the monopolistic competition model:

$$f_i^d + x_i^d(\omega) = A_i^d(\ell_i^d(\omega))^\alpha Q_i^u(\omega)^{1-\alpha}, \qquad \omega \in [0, M_i^d], \quad \alpha \in [0,1], \quad i \in \{H, F\}.$$

Imposing symmetry and (B.15), we obtain

$$M_i^d = \frac{A_i^d}{\sigma f_i^d}(M_i^d \ell_i^d(\omega))^\alpha \left(M_i^d Q_i^u\right)^{1-\alpha}$$

or

$$X_i^d \equiv \left(M_i^d\right)^{\frac{\sigma}{\sigma-1}} x_i^d = \bar{A}_i^d \left[\left(L_i^d\right)^\alpha \left(\left(L_i^d\right)^\alpha \left(\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}\right)^{1-\alpha}\right]^{\frac{\sigma}{\sigma-1}}. \tag{B.21}$$

where

$$\bar{A}_i^d \equiv \left(\frac{A_i^d}{\sigma f_i^d}\right)^{\frac{\sigma}{\sigma-1}} (\sigma-1) f_i^d.$$

This aggregate output X_i^d is sold domestically or exported, and thus

$$\bar{A}_i^d \left(\left(L_i^d\right)^\alpha \left(\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}}\right)^{\gamma^d} = Q_{ii}^d + Q_{ij}^d,$$

where

$$\gamma^d \equiv 1/(\sigma-1).$$

In sum, starting from the monopolistic competition model, we have derived equation (15) in the external economies model.

Trade Balance Consider next the trade balance condition. Starting from the monopolistic competition economy, we have

$$\frac{p_{ji}^d}{1+t_i^d} M_j^d q_{ji}^d + \frac{p_{ji}^u}{1+t_i^u} M_i^d M_j^u q_{ji}^u = \frac{\tilde{p}_{ij}^d}{1-v_i^d} M_i^d q_{ij}^d + \frac{\tilde{p}_{ij}^u}{1-v_i^u} M_j^d M_i^u q_{ij}^u, \tag{B.22}$$

which equates the import revenue in i paid to exporters in j with export revenue collected from j by producers in i.

Now from equations (B.5) and (B.6), notice that we have

$$\frac{p_{ji}^d(\omega)\, q_{ji}^d(\omega)}{P_{ji}^d Q_{ji}^d} = \left(\frac{p_{ji}^d(\omega)}{P_{ji}^d}\right)^{1-\sigma},$$

and

$$P_{ji}^d = \left[\int_0^{M_j^d} \left(p_{ji}^d(\omega)\right)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}},$$

so given symmetry, we have

$$P_{ji}^d = \left(M_j^d\right)^{\frac{1}{1-\sigma}} p_{ji}^d \tag{B.23}$$

and

$$P^d_{ji}Q^d_{ji} = \left(M^d_j\right)^{\frac{-1}{\sigma-1}} p^d_{ji} \times \left(M^d_i\right)^{\frac{\sigma}{\sigma-1}} q^d_{ii} = M^d_j p^d_{ji} q^d_{ii}.$$

Similarly, for inputs

$$P^u_{ji}Q^u_{ji} = \left(M^u_j\right)^{\frac{1}{1-\theta}} p^u_{ji} \times M^d_i \left(M^u_i\right)^{\frac{\theta}{\theta-1}} q^u_{ji} = p^u_{ji} M^d_i M^u_j q^u_{ji}.$$

This implies that we can write total imports in the trade balance condition (B.22) as

$$\frac{P^d_{ji}}{1+t^d_i}Q^d_{ji} + \frac{P^u_{ji}}{1+t^u_i}Q^u_{ji} = \bar{P}^d_{ji}Q^d_{ji} + \bar{P}^u_{ji}Q^u_{ji},$$

which corresponds to the left-hand-side of the trade balance condition (17) for the economy with external economies of scale after noting that $\bar{P}^d_{ji}$ and $\bar{P}^u_{ji}$ are the prices collected by country j (or Foreign) exporters (not those paid by domestic or country i consumers).

Now consider revenue from exporting final goods. Notice that, regardless of whether the Foreign government imposes import tariffs or not, we have that export revenue is

$$\frac{\tilde{p}^d_{ij}}{1-v^d_i}M^d_i q^d_{ij} + \frac{\tilde{p}^u_{ij}}{1-v^u_i}M^d_j M^u_i q^u_{ij}$$

Prices paid by country j are $\tilde{p}^d_{ij}/\left(1-v^d_i\right)$ and $\tilde{p}^u_{ij}/\left(1-v^u_i\right)$, so following analogous steps, the right-hand-side of (17) becomes

$$\frac{\tilde{P}^d_{ij}}{1-v^d_i}Q^d_{ij} + \frac{\tilde{P}^u_{ij}}{1-v^u_i}Q^u_{ij} = \bar{P}^d_{ij}Q^d_{ij} + \bar{P}^u_{ij}Q^u_{ij},$$

where $\bar{P}^d_{ij}$ and $\bar{P}^u_{ij}$ are the prices paid by country j (or Foreign) importers (not those paid collected by country i exporters).

Note: In the main text, we denote $\bar{P}^d_{ji}$, $\bar{P}^u_{ji}$, $\bar{P}^d_{ij}$, and $\bar{P}^u_{ij}$ as simply P^d_{ji}, P^u_{ji}, P^d_{ij}, and P^u_{ij}. We do so not to clutter the notation, but these are distinct from the price indices applying to the monopolistic competition model, which are always built based on prices paid by consumers, regardless of their country.

Optimality Conditions We have so far shown that the four 'resource' constraints (14) through (17) of our isomorphic economy can be derived from our baseline model with monopolistic competition and internal economies of scale. We next turn to an analogous derivation for the optimality conditions (18) through (21).

Given our functional forms for utility and technology, these optimality conditions in the model with external economies of scale are given by

$$\left(\frac{Q^d_{ii}}{Q^d_{ji}}\right)^{-\frac{1}{\sigma}} = \frac{\left(1-v^d_i\right)}{\left(1+\tau^d_i\right)}\frac{\bar{P}^d_{ij}}{\bar{P}^d_{ji}}; \tag{B.24}$$

$$\left(\frac{Q^u_{ii}}{Q^u_{ji}}\right)^{-\frac{1}{\theta}} = \frac{\left(1-v^u_i\right)}{\left(1+\tau^u_i\right)}\frac{\bar{P}^u_{ij}}{\bar{P}^u_{ji}}; \tag{B.25}$$

$$(1-\alpha)\hat{A}^d_i\left(L^d_i\right)^{\alpha}\left(\left(Q^u_{ii}\right)^{\frac{\theta-1}{\theta}} + \left(Q^u_{ji}\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}}\frac{1}{Q^u_{ii}}\frac{\left(Q^u_{ii}\right)^{\frac{\theta-1}{\theta}}}{\left(Q^u_{ii}\right)^{\frac{\theta-1}{\theta}} + \left(Q^u_{ji}\right)^{\frac{\theta-1}{\theta}}} = \frac{\left(1-v^u_i\right)}{\left(1-v^d_i\right)}\frac{\bar{P}^u_{ij}}{\bar{P}^d_{ij}}; \tag{B.26}$$

$$(1-\alpha)\hat{A}^u_i\frac{1}{Q^u_{ii}}\frac{\left(Q^u_{ii}\right)^{\frac{\theta-1}{\theta}}}{\left(Q^u_{ii}\right)^{\frac{\theta-1}{\theta}} + \left(Q^u_{ji}\right)^{\frac{\theta-1}{\theta}}} = \alpha\frac{1}{L^d_i}. \tag{B.27}$$

Optimal in Final-Good Consumption Let us begin with the first one, equating the marginal rate of substitution in final-good consumption to relative prices. Given equation (B.7) in the model with monopolistic competition, we have

$$\frac{Q_{ii}^d}{Q_{ji}^d} = \left(\frac{P_{ii}^d}{P_{ji}^d}\right)^{-\sigma},$$

where Q_{ji}^d, Q_{ji}^d, P_{ji}^d and P_{ji}^d are defined in (B.18) and (B.23). Thus, we have

$$\left(\frac{Q_{ii}^d}{Q_{ji}^d}\right)^{-\frac{1}{\sigma}} = \frac{P_{ii}^d}{P_{ji}^d} = \frac{\left(1 - v_i^d\right) \bar{P}_{ij}^d}{\left(1 + \tau_i^d\right) \bar{P}_{ji}^d},$$

where $P_{ii}^d = \left(1 - v_i^d\right) \bar{P}_{ij}^d$ because of the indifference between selling domestically or exporting to country j (remember that, in the external economies of scale model, $\bar{P}_{ij}^d$ is the price paid by consumers in j for final goods from j). We have thus derived equation (B.24), which corresponds to (18) in the external economies of scale model.

Optimal in Input Consumption The derivation of equation (B.25), equating the marginal rate of substitution in input consumption to relative prices, is completely analogous. In particular, from equation (B.10) in the model with monopolistic competition, we have

$$\frac{Q_{ii}^u}{Q_{ji}^u} = \left(\frac{P_{ii}^u}{P_{ji}^u}\right)^{-\theta},$$

where Q_{ji}^u, Q_{ji}^u, P_{ji}^u and P_{ji}^u are defined in (B.18) and (B.23). Thus, we have

$$\left(\frac{Q_{ii}^u}{Q_{ji}^u}\right)^{-\frac{1}{\theta}} = \frac{P_{ii}^u}{P_{ji}^u} = \frac{\left(1 - v_i^u\right) \bar{P}_{ij}^u}{\left(1 + \tau_i^u\right) \bar{P}_{ji}^u},$$

where $P_{ii}^u = \left(1 - v_i^d\right) \bar{P}_{ij}^u$ because of the indifference between selling domestically or exporting to country j (remember that, in the external economies of scale model, $\bar{P}_{ij}^u$ is the price paid by consumers in j for final goods from j). We have thus derived equation (B.25), which corresponds to (19) in the external economies of scale model.

Optimality Domestic Input Allocation We next move to the third optimality condition (20), which equates the benefits of exporting domestic intermediate inputs with the benefits of using those additional domestic inputs to produce an additional amount of the final good that is in turn exported.

We begin with equation (B.11), and note that aggregate input use in country i in the monopolistic competition model is given by

$$P_i^u Q_i^u = (1 - \alpha) \left(\tilde{p}_{ii}^d q_{ii}^d + \frac{1 - v_i^d}{1 + t_j^d} \tilde{p}_{ij}^d q_{ij}^d\right) M_i^d. \tag{B.28}$$

To reiterate this, note from (B.2) that $\frac{1-v_i^d}{1+t_j^d}\tilde{p}_{ij}^d = \tau^d p_{ii}^d$, and plugging in equation (B.13), we obtain

$$P_i^u Q_i^u = (1 - \alpha)\, p_{ii}^d \left(q_{ii}^d + \tau^d q_{ij}^d\right) M_i^d = (1 - \alpha)\, p_{ii}^d x_i^d M_i^d. \tag{B.29}$$

Next invoke equation (B.10) applied to $P^u_{ii}Q^u_{ii}$ to obtain (after plugging in (B.28)):

$$P^u_i Q^u_i = P^u_{ii} Q^u_{ii} \left(\frac{P^u_{ii}}{P^u_i}\right)^{\theta-1}. \tag{B.30}$$

Combining (B.29) and (B.30), we obtain:

$$(1-\alpha)\, p^d_{ii} x^d_i M^d_i = P^u_{ii} Q^u_{ii} \left(\frac{P^u_{ii}}{P^u_i}\right)^{\theta-1},$$

which we decompose as

$$(1-\alpha) \times \left(M^d_i\right)^{\frac{-1}{\sigma-1}} p^d_{ii} \times \left(M^d_i\right)^{\frac{\sigma}{\sigma-1}} x^d_i = P^u_{ii} Q^u_{ii} \left(\frac{P^u_{ii}}{P^u_i}\right)^{\theta-1}, \tag{B.31}$$

Now remember from equation (B.21) derived above that

$$\left(M^d_i\right)^{\frac{\sigma}{\sigma-1}} x^d_i = \bar{A}^d_i \left[\left(L^d_i\right)^{\alpha} \left(\left(L^d_i\right)^{\alpha} \left(\left(Q^u_{ii}\right)^{\frac{\theta-1}{\theta}} + \left(Q^u_{ji}\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}\right)^{1-\alpha}\right]^{\frac{\sigma}{\sigma-1}},$$

and also from (B.23) that $\left(M^d_i\right)^{\frac{-1}{\sigma-1}} p^d_{ii} = P^d_{ii}$, so we can write (B.31) as

$$P^d_{ii}\,(1-\alpha)\,\hat{A}^d_i \left(L^d_i\right)^{\alpha} \left(\left(Q^u_{ii}\right)^{\frac{\theta-1}{\theta}} + \left(Q^u_{ji}\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}} \frac{1}{Q^u_{ii}} = P^u_{ii} \left(\frac{P^u_{ii}}{P^u_i}\right)^{\theta-1}.$$

Now invoke (B.10)

$$\frac{Q^u_{ii}}{Q^u_i} = \left(\frac{P^u_{ii}}{P^u_i}\right)^{-\theta},$$

to obtain

$$P^d_{ii}\,(1-\alpha)\,\hat{A}^d_i \left(L^d_i\right)^{\alpha} \left(\left(Q^u_{ii}\right)^{\frac{\theta-1}{\theta}} + \left(Q^u_{ji}\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}} \frac{1}{Q^u_{ii}} = P^u_{ii} \left(\frac{Q^u_{ii}}{Q^u_i}\right)^{-\frac{\theta-1}{\theta}},$$

which given the definition of Q^u_i in (B.1) delivers

$$P^d_{ii}\,(1-\alpha)\,\hat{A}^d_i \left(L^d_i\right)^{\alpha} \left(\left(Q^u_{ii}\right)^{\frac{\theta-1}{\theta}} + \left(Q^u_{ji}\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}} \frac{1}{Q^u_{ii}} \frac{\left(Q^u_{ii}\right)^{\frac{\theta-1}{\theta}}}{\left(Q^u_{ii}\right)^{\frac{\theta-1}{\theta}} + \left(Q^u_{ji}\right)^{\frac{\theta-1}{\theta}}} = P^u_{ii}.$$

The final step is to note, as we did above, that indifference between selling domestically and exporting, delivers $P^d_{ii} = \left(1-v^d_i\right)\bar{P}^d_{ij}$ and $P^u_{ii} = \left(1-v^d_i\right)\bar{P}^u_{ij}$, where remember that $\bar{P}^d_{ij}$ and $\bar{P}^u_{ij}$ are the prices paid by country j residents. In sum, we have derived equation (B.26), or

$$(1-\alpha)\,\hat{A}^d_i \left(L^d_i\right)^{\alpha} \left(\left(Q^u_{ii}\right)^{\frac{\theta-1}{\theta}} + \left(Q^u_{ji}\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}} \frac{1}{Q^u_{ii}} \frac{\left(Q^u_{ii}\right)^{\frac{\theta-1}{\theta}}}{\left(Q^u_{ii}\right)^{\frac{\theta-1}{\theta}} + \left(Q^u_{ji}\right)^{\frac{\theta-1}{\theta}}} = \frac{\left(1-v^d_i\right)\bar{P}^u_{ij}}{\left(1-v^d_i\right)\bar{P}^d_{ij}}.$$

Optimal Labor Market Allocation We finally tackle the fourth optimality condition, associated with the optimal allocation of labor across sectors. We begin with the firm-level monopolistic competition model, equating the wage paid in both sectors. Because of free entry, total revenue upstream must equal total wage payments, while in the downstream sector, wage payments are a share α of total revenue, as indicated in equation (B.9), or

$$\frac{\alpha\left(\tilde{p}_{ii}^{d}q_{ii}^{d}+\frac{1-v_i^d}{1+t_j^d}\tilde{p}_{ij}^{d}q_{ij}^{d}\right)}{\ell_i^d}=\frac{\tilde{p}_{ii}^{u}M_i^d q_{ii}^{u}+\frac{1-v_i^u}{1+t_j^u}\tilde{p}_{ij}^{u}M_j^d q_{ij}^{u}}{\ell_i^u}.$$

Now noting that from (B.2), we have $\frac{1-v_i^d}{1+t_j^d}\tilde{p}_{ij}^d=\tau^d p_{ii}^d$ (and analogously $\frac{1-v_i^u}{1+t_j^u}\tilde{p}_{ij}^u=\tau^u p_{ii}^u$), and plugging in equations (B.13) and (B.14), we have

$$\frac{\alpha p_{ii}^d x_i^d}{\ell_i^d}=\frac{p_{ii}^u x_i^u}{\ell_i^u}. \tag{B.32}$$

Next, invoke the price index definitions – see equation B.23) – as well as the definitions $L_i^d=M_i^d\ell_i^d$ and $L_i^u=M_i^u\ell_i^u$, to write (B.32) as

$$\alpha P_{ii}^d\frac{x_i^d\left(M_i^d\right)^{\frac{\sigma}{\sigma-1}}}{L_i^d}=P_{ii}^u\frac{\left(M_i^u\right)^{\frac{\theta}{\theta-1}}x_i^u}{L_i^u}.$$

Next, plugging (B.20) and (B.21), delivers

$$\frac{\alpha P_{ii}^d}{L_i^d}\hat{A}_i^d\left(L_i^d\right)^{\alpha}\left(\left(L_i^d\right)^{\alpha}\left(\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}}+\left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}\right)^{1-\alpha}=P_{ii}^u\hat{A}_i^u,$$

where remember that $\hat{A}_i^d$ and $\hat{A}_i^u$ are defined in equations (12) and (13) in the main text.

The next step is to note, as we did above, that indifference between selling domestically and exporting, delivers $P_{ii}^d=\left(1-v_i^d\right)\bar{P}_{ij}^d$ and $P_{ii}^u=\left(1-v_i^d\right)\bar{P}_{ij}^u$, where remember that $\bar{P}_{ij}^d$ and $\bar{P}_{ij}^u$ are the prices paid by country j residents, so we have

$$\frac{\alpha}{L_i^d}\hat{A}_i^d\left(L_i^d\right)^{\alpha}\left(\left(L_i^d\right)^{\alpha}\left(\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}}+\left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}\right)^{1-\alpha}=\hat{A}_i^u\frac{\left(1-v_i^d\right)}{\left(1-v_i^d\right)}\frac{\bar{P}_{ij}^u}{\bar{P}_{ij}^d}.$$

The final step is to plug optimality condition (B.26) and cancel terms to obtain

$$\frac{\alpha}{L_i^d}=\left(1-\alpha\right)\hat{A}_i^u\frac{1}{Q_{ii}^u}\frac{\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}}}{\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}}+\left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}},$$

which corresponds to the last optimality condition (B.26).

This completes the proof of the isomorphism claimed in Proposition 3.

C Optimal Trade Policy for a Small Open Economy with No Domestic Distortions: Derivations

C.1 First-Best Policies

We begin by characterizing the solution to the program

$$\begin{aligned} \max \quad & U\left(Q_{HH}^d, Q_{FH}^d\right) \\ s.t. \quad & \hat{A}_H^u L_H = Q_{HH}^u + Q_{HF}^u \\ & \hat{A}_H^d F^d\left(Q_{HH}^u, Q_{FH}^u\right) = Q_{HH}^d + Q_{HF}^d \\ & P_{FH}^d Q_{FH}^d + P_{FH}^u Q_{FH}^u = Q_{HF}^d (Q_{HF}^d)^{-\frac{1}{\sigma}} P_{FF}^d \left(Q_{FF}^d\right)^{\frac{1}{\sigma}} + Q_{HF}^u \left(Q_{HF}^u\right)^{-\frac{1}{\theta}} P_{FF}^u \left(Q_{FF}^u\right)^{\frac{1}{\theta}}, \end{aligned}$$

where $\hat{A}_H^u$ and $\hat{A}_H^d$ are given by

$$\hat{A}_H^d = \bar{A}_H^d \left(F^d\left(Q_{HH}^u, Q_{FH}^u\right)\right)^{\gamma^d}$$

and

$$\hat{A}_H^u = \bar{A}_H^u \left(L_H\right)^{\gamma^u},$$

respectively. We also note that

$$U\left(Q_{HH}^d, Q_{FH}^d\right) = \left(\left(Q_{HH}^d\right)^{\frac{\sigma-1}{\sigma}} + \left(Q_{FH}^d\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

and that

$$F^d\left(Q_{HH}^u, Q_{FH}^u\right) = \left(\left(Q_{HH}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{FH}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}},$$

but this will prove immaterial for the derivations below.

We first write the Lagrangian of this problem

$$\begin{aligned} & U\left(Q_{HH}^d, Q_{FH}^d\right) + \mu_u \left[\bar{A}_H^u \left(L_H\right)^{1+\gamma^u} - Q_{HH}^u - Q_{HF}^u\right] + \mu_d \left[\bar{A}_H^d \left(F^d\left(Q_{HH}^u, Q_{FH}^u\right)\right)^{1+\gamma^d} - Q_{HH}^d - Q_{HF}^d\right] \\ & + \mu_{TB} \left[Q_{HF}^d (Q_{HF}^d)^{-\frac{1}{\sigma}} P_{FF}^d \left(Q_{FF}^d\right)^{\frac{1}{\sigma}} + Q_{HF}^u \left(Q_{HF}^u\right)^{-\frac{1}{\theta}} P_{FF}^u \left(Q_{FF}^u\right)^{\frac{1}{\theta}} - P_{FH}^d Q_{FH}^d - P_{FH}^u Q_{FH}^u\right]. \end{aligned}$$

The first order conditions associated with the choices of Q_{HH}^d, Q_{FH}^d, Q_{HF}^d, Q_{HH}^u, Q_{FH}^u, and Q_{HF}^u are as follows:

$$U_{Q_{HH}^d}\left(Q_{HH}^d, Q_{FH}^d\right) = \mu_d \tag{C.1}$$

$$U_{Q_{FH}^d}\left(Q_{HH}^d, Q_{FH}^d\right) = \mu_{TB} P_{FH}^d \tag{C.2}$$

$$\mu_d = \mu_{TB} \frac{\sigma-1}{\sigma} P_{HF}^d \tag{C.3}$$

$$\mu_u = \mu_d \left(1+\gamma^d\right) \bar{A}_H^d \left(F^d\left(Q_{HH}^u, Q_{FH}^u\right)\right)^{\gamma^d} F_{Q_{HH}^u}^d \left(Q_{HH}^u, Q_{FH}^u\right) \tag{C.4}$$

$$\mu_{TB} P_{FH}^u = \mu_d \left(1+\gamma^d\right) \bar{A}_H^d \left(F^d\left(Q_{HH}^u, Q_{FH}^u\right)\right)^{\gamma^d} F_{Q_{FH}^u}^d \left(Q_{HH}^u, Q_{FH}^u\right) \tag{C.5}$$

$$\mu_u = \mu_{TB} \frac{\theta-1}{\theta} P_{HF}^u \tag{C.6}$$

Dividing equation (23) by equation (C.2), and plugging in (C.3), we obtain obtain:

$$\frac{U_{Q_{HH}^d}\left(Q_{HH}^d, Q_{FH}^d\right)}{U_{Q_{FH}^d}\left(Q_{HH}^d, Q_{FH}^d\right)} = \frac{\frac{\sigma-1}{\sigma} P_{HF}^d}{P_{FH}^d},$$

which corresponds to the first optimality condition (23) in the main text.

Next, we divide equation (C.4) by equation (C.5), and plugging in (C.6), delivers

$$\frac{F_{Q_{HH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right)}{F_{Q_{FH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right)} = \frac{\frac{\theta-1}{\theta} P_{HF}^u}{P_{FH}^u},$$

which corresponds to the second optimality condition (24) in the main text.

Finally, combining equation (C.4) with the ratio of equations (C.3) and (C.6) produces

$$\left(1+\gamma^d\right) \bar{A}_H^d \left(F^d\left(Q_{HH}^u, Q_{FH}^u\right)\right)^{\gamma^d} F_{Q_{HH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right) = \frac{\frac{\theta-1}{\theta}}{\frac{\sigma-1}{\sigma}} \frac{P_{HF}^u}{P_{HF}^d},$$

which corresponds to the third optimality condition (25) in the main text.

C.2 Generalizations

As demonstrated in the derivations in the above Appendix C.1, we have made no use of the properties of the functions $U\left(Q_{HH}^d, Q_{FH}^d\right)$ and $F^d\left(Q_{HH}^u, Q_{FH}^u\right)$. In particular, we could assume that

$$U\left(Q_{HH}^d, Q_{FH}^d\right) = \left(\left(Q_{HH}^d\right)^{\frac{\sigma_H-1}{\sigma_H}} + \left(Q_{FH}^d\right)^{\frac{\sigma_H-1}{\sigma_H}}\right)^{\frac{\sigma_H}{\sigma_H-1}}$$

and that

$$F^d\left(Q_{HH}^u, Q_{FH}^u\right) = \left(\left(Q_{HH}^u\right)^{\frac{\theta_H-1}{\theta_H}} + \left(Q_{FH}^u\right)^{\frac{\theta_H-1}{\theta_H}}\right)^{\frac{\theta_H}{\theta_H-1}},$$

with potentially $\sigma_H \neq \sigma$ and $\theta_H \neq \theta$. It is clear from the derivations in section 4.1 that the first-best trade policies will continue to satisfy

$$\begin{aligned}
1+t_H^d &= \left(1+\gamma^d\right)\left(1+\bar{T}\right); \\
1+t_H^u &= 1+\bar{T}; \\
1-v_H^d &= \frac{\sigma-1}{\sigma}\left(1+\gamma^d\right)\left(1+\bar{T}\right); \\
1-v_H^u &= \frac{\theta-1}{\theta}\left(1+\bar{T}\right).
\end{aligned}$$

The only significant difference in this case is that if we want to invoke our isomorphism to claim that these policies also implement the first best in the Krugman vertical economy with internal economies of scale, then we necessarily need to impose $\gamma^d = 1/\left(\sigma_H - 1\right)$, and thus the level of the tariff escalation is closely related to the degree of differentiation in the final-good sector. This is not particularly surprising, since love-for-variety effects will be more powerful, the lower the degree of substitutability across final goods.

C.3 Alternative First-Best Implementations

In this Appendix, we explore the structure of first-best policies when the set of available instruments includes domestic production subsidies, domestic consumption subsidies, or domestic production/consumption subsidies that only apply to domestic transactions.

C.3.1 Discriminatory Domestic Subsidies

We first consider the case in which the Home government has access to discriminatory domestic subsidies s_{HH}^d and s_{HH}^u that apply only to purchases of final goods and of intermediate inputs involving only Home residents. The inclusion of these instruments alters the decentralized market equilibrium conditions (18), (19) and (20) as follows:

$$\begin{aligned}
\frac{U_{Q_{HH}^d}\left(Q_{HH}^d, Q_{FH}^d\right)}{U_{Q_{FH}^d}\left(Q_{HH}^d, Q_{FH}^d\right)} &= \left(1 - s_{HH}^d\right) \frac{\left(1 - v_H^d\right)}{\left(1 + t_H^d\right)} \frac{P_{HF}^d}{P_{FH}^d}; \\
\frac{F_{Q_{HH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right)}{F_{Q_{FH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right)} &= \left(1 - s_{HH}^u\right) \frac{\left(1 - v_H^u\right)}{\left(1 + t_H^u\right)} \frac{P_{HF}^u}{P_{FH}^u}; \\
\hat{A}_H^d F_{Q_{HH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right) &= \left(1 - s_{HH}^u\right) \frac{\left(1 - v_H^u\right)}{\left(1 - v_H^d\right)} \frac{P_{HF}^u}{P_{HF}^d}.
\end{aligned}$$

Comparing these equations to those characterizing the optimal allocations, that is equations (23), (24), and (25), it is clear that the first best can be achieved by setting

$$\begin{aligned}
\left(1 + t_H^d\right)\left(1 - s_{HH}^d\right) &= \left(1 + \gamma^d\right)\left(1 + \bar{T}\right); \\
1 + t_H^u &= 1 + \bar{T}; \\
1 - v_H^d &= \frac{\sigma - 1}{\sigma}\left(1 + \gamma^d\right)\left(1 + \bar{T}\right); \\
\left(1 - s_{HH}^u\right)\left(1 - v_H^u\right) &= \frac{\theta - 1}{\theta}\left(1 + \bar{T}\right).
\end{aligned}$$

These equations illustrate that a downstream discriminatory subsidy is a perfect substitute for the downstream import tariff (only the product $\left(1 + t_H^d\right)\left(1 - s_{HH}^d\right)$ matters), while an upstream discriminatory subsidy is a perfect substitute for the upstream export tax (only the product $\left(1 - s_{HH}^u\right)\left(1 - v_H^u\right)$ matters). A straightforward implication of this result is that, whenever $1+\gamma^d = \sigma/\left(\sigma - 1\right)$, as imposed by our isomorphism, the first best can be attained by setting $\left(1 + t_H^d\right)\left(1 - s_{HH}^d\right) = \sigma/\left(\sigma - 1\right)$ and $\left(1 - s_{HH}^u\right)\left(1 - v_H^u\right) = \left(\theta - 1\right)/\theta$. Thus, the first best can be achieved with only discriminatory subsidies, or with a combination of a subsidy in one sector and a trade instrument in the other sector. When only domestic instruments are used, we necessarily have $s_{HH}^d = 1/\sigma$ and $s_{HH}^u = 1/\theta$. Whether or not the resulting first-best policies entail tariff escalation depends on the level of the downstream domestic subsidy, since $\left(1 + t_H^d\right)/\left(1 + t_H^u\right) = \left(1 + \gamma^d\right)/\left(1 - s_{HH}^d\right)$.

C.3.2 Production Subsidies

We next consider the case of production subsidies s_H^d and s_H^u that apply to Home production of final goods and intermediate inputs, regardless of where those goods are sold (domestically or exported). The inclusion of these instruments alters the decentralized market equilibrium conditions (18), (19) and (20) as follows:

$$\begin{aligned}\frac{U_{Q_{HH}^d}\left(Q_{HH}^d,Q_{FH}^d\right)}{U_{Q_{FH}^d}\left(Q_{HH}^d,Q_{FH}^d\right)} &= \frac{\left(1-v_H^d\right)}{\left(1+t_H^d\right)}\frac{P_{HF}^d}{P_{FH}^d};\\ \frac{F_{Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F_{Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)} &= \frac{\left(1-v_H^u\right)}{\left(1+t_H^u\right)}\frac{P_{HF}^u}{P_{FH}^u};\\ \hat{A}_H^d F_{Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right) &= \left(1-s_H^d\right)\frac{\left(1-v_H^u\right)}{\left(1-v_H^d\right)}\frac{P_{HF}^u}{P_{HF}^d}.\end{aligned}$$

Comparing these equations to those characterizing the optimal allocations, that is equations (23), (24), and (25), it is clear that the first best can be achieved by setting

$$\begin{aligned}1+t_H^d &= \left(1-s_H^d\right)\left(1+\gamma^d\right)\left(1+\bar{T}\right);\\ 1+t_H^u &= 1+\bar{T};\\ 1-v_H^d &= \frac{\sigma-1}{\sigma}\left(1+\gamma^d\right)\left(1-s_H^d\right)\left(1+\bar{T}\right);\\ 1-v_H^u &= \frac{\theta-1}{\theta}\left(1+\bar{T}\right).\end{aligned}$$

Notice that, as long as $s_H^d>0$, the set of first-best policies will entail this subsidy and at least two additional trade instruments. For instance, when setting $s_H^d=\gamma^d/\left(1+\gamma^d\right)$, the first best can be achieved with this production subsidy and two export taxes ($1-v_H^d=(\sigma-1)/\sigma$ and $1-v_H^u=(\theta-1)/\theta$), while setting all import tariffs to zero. Alternatively, when setting $s_H^d=(1+\gamma^u)/\gamma^u=(\theta-1)/\theta$, the first best can be achieved with this production subsidy and two import tariffs ($t_H^d=1/(\sigma-1)$ and $t_H^u=1/(\theta-1)$).

Regardless of the actual implementation, the tariff escalation wedge is given by:

$$\frac{1+t_H^d}{1+t_H^u}=\left(1-s_H^d\right)\left(1+\gamma^d\right).$$

C.3.3 Consumption Subsidies

We finally consider the case of consumption subsidies s_H^d and s_H^u that apply to Home consumption of final goods and of intermediate inputs, regardless of where those goods are purchased (domestically or imported). The inclusion of these instruments alters the decentralized market equilibrium conditions (18), (19) and (20) as follows:

$$\begin{aligned}\frac{U_{Q_{HH}^d}\left(Q_{HH}^d,Q_{FH}^d\right)}{U_{Q_{FH}^d}\left(Q_{HH}^d,Q_{FH}^d\right)} &= \frac{\left(1-v_H^d\right)}{\left(1+t_H^d\right)}\frac{P_{HF}^d}{P_{FH}^d};\\ \frac{F_{Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F_{Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)} &= \frac{\left(1-v_H^u\right)}{\left(1+t_H^u\right)}\frac{P_{HF}^u}{P_{FH}^u};\\ \hat{A}_H^d F_{Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right) &= \left(1-s_H^u\right)\frac{\left(1-v_H^u\right)}{\left(1-v_H^d\right)}\frac{P_{HF}^u}{P_{HF}^d}.\end{aligned}$$

These equations are completely analogous to those applying to the case of prouduction subsidies, with s_H^u replacing s_H^d, so the conclusions that arise from it are also analogous.

C.4 Second-Best Import Tariffs

In this Appendix we characterize the second-best import tariffs when the government only has access to import tariffs upstream and downstream.

A. Second-Best Import Tariffs with Scale Economies

As mentioned in the main text, the second-best optimal allocation will seek to solve the same problem laid out in section 4.1 expanded to include the additional constraint:

$$\hat{A}_H^d F_{Q_{HH}^u}^d \left(Q_{HH}^u, Q_{FH}^u\right) = \frac{P_{HF}^d}{P_{HF}^u} = \frac{\left(Q_{HF}^u\right)^{-\frac{1}{\theta}} P_{FF}^u \left(Q_{FF}^u\right)^{\frac{1}{\theta}}}{\left(Q_{HF}^d\right)^{-\frac{1}{\sigma}} P_{FF}^d \left(Q_{FF}^d\right)^{\frac{1}{\sigma}}}.$$

More specifically, the planner problem is now

$$\begin{aligned}
\max \quad & U\left(Q_{HH}^d, Q_{FH}^d\right) \\
s.t. \quad & \hat{A}_H^u L_H = Q_{HH}^u + Q_{HF}^u \\
& \hat{A}_H^d F^d\left(Q_{HH}^u, Q_{FH}^u\right) = Q_{HH}^d + Q_{HF}^d \\
& P_{FH}^d Q_{FH}^d + P_{FH}^u Q_{FH}^u = Q_{HF}^d \left(Q_{HF}^d\right)^{-\frac{1}{\sigma}} P_{FF}^d \left(Q_{FF}^d\right)^{\frac{1}{\sigma}} + Q_{HF}^u \left(Q_{HF}^u\right)^{-\frac{1}{\theta}} P_{FF}^u \left(Q_{FF}^u\right)^{\frac{1}{\theta}} \\
& \hat{A}_H^d F_{Q_{HH}^u}^d \left(Q_{HH}^u, Q_{FH}^u\right) = \frac{\left(Q_{HF}^u\right)^{-\frac{1}{\theta}} P_{FF}^u \left(Q_{FF}^u\right)^{\frac{1}{\theta}}}{\left(Q_{HF}^d\right)^{-\frac{1}{\sigma}} P_{FF}^d \left(Q_{FF}^d\right)^{\frac{1}{\sigma}}}
\end{aligned}$$

where $\hat{A}_H^u$ and $\hat{A}_H^d$ are given by

$$\hat{A}_H^d = \bar{A}_H^d \left(F^d\left(Q_{HH}^u, Q_{FH}^u\right)\right)^{\gamma^d}$$

and

$$\hat{A}_H^u = \bar{A}_H^u \left(L_H\right)^{\gamma^u},$$

respectively.

We first write the Lagrangian of this problem

$$\begin{aligned}
& U\left(Q_{HH}^d, Q_{FH}^d\right) + \mu_u \left[\bar{A}_H^u \left(L_H\right)^{1+\gamma^u} - Q_{HH}^u - Q_{HF}^u\right] + \mu_d \left[\bar{A}_H^d \left(F^d\left(Q_{HH}^u, Q_{FH}^u\right)\right)^{1+\gamma^d} - Q_{HH}^d - Q_{HF}^d\right] \\
& + \mu_{TB} \left[Q_{HF}^d \left(Q_{HF}^d\right)^{-\frac{1}{\sigma}} P_{FF}^d \left(Q_{FF}^d\right)^{\frac{1}{\sigma}} + Q_{HF}^u \left(Q_{HF}^u\right)^{-\frac{1}{\theta}} P_{FF}^u \left(Q_{FF}^u\right)^{\frac{1}{\theta}} - P_{FH}^d Q_{FH}^d - P_{FH}^u Q_{FH}^u\right] \\
& + \mu_{SB} \left[\hat{A}_H^d F_{Q_{HH}^u}^d \left(Q_{HH}^u, Q_{FH}^u\right) - \frac{\left(Q_{HF}^u\right)^{-\frac{1}{\theta}} P_{FF}^u \left(Q_{FF}^u\right)^{\frac{1}{\theta}}}{\left(Q_{HF}^d\right)^{-\frac{1}{\sigma}} P_{FF}^d \left(Q_{FF}^d\right)^{\frac{1}{\sigma}}}\right]
\end{aligned}$$

The first order conditions associated with the choices of Q_{HH}^d, Q_{FH}^d, Q_{HF}^d, Q_{HH}^u, Q_{FH}^u, and Q_{HF}^u are as follows:

$$
\begin{aligned}
U_{Q^d_{HH}}\left(Q^d_{HH},Q^d_{FH}\right) &= \mu_d && \text{(C.7)}\\
U_{Q^d_{FH}}\left(Q^d_{HH},Q^d_{FH}\right) &= \mu_{TB}P^d_{FH} && \text{(C.8)}\\
\mu_d &= \mu_{TB}\frac{\sigma-1}{\sigma}P^d_{HF}-\mu_{SB}\frac{1}{\sigma}\frac{1}{Q^d_{HF}}\frac{P^u_{HF}}{P^d_{HF}} && \text{(C.9)}\\
\mu_u &= \mu_d\left(1+\gamma^d\right)\bar{A}^d_H\left(F^d\left(Q^u_{HH},Q^u_{FH}\right)\right)^{\gamma^d}F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)\\
&\quad+\mu_{SB}\bar{A}^d_H\left(F^d\left(Q^u_{HH},Q^u_{FH}\right)\right)^{\gamma^d}F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)\\
&\quad\times\left[\gamma^d\frac{F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)}{F^d\left(Q^u_{HH},Q^u_{FH}\right)}+\frac{F^d_{Q^u_{HH},Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)}{F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)}\right] && \text{(C.10)}\\
\mu_{TB}P^u_{FH} &= \mu_d\left(1+\gamma^d\right)\bar{A}^d_H\left(F^d\left(Q^u_{HH},Q^u_{FH}\right)\right)^{\gamma^d}F^d_{Q^u_{FH}}\left(Q^u_{HH},Q^u_{FH}\right)\\
&\quad+\mu_{SB}\bar{A}^d_H\left(F^d\left(Q^u_{HH},Q^u_{FH}\right)\right)^{\gamma^d}F^d_{Q^u_{FH}}\left(Q^u_{HH},Q^u_{FH}\right)\\
&\quad\times\left[\gamma^d\frac{F^d_{Q^u_{FH}}\left(Q^u_{HH},Q^u_{FH}\right)}{F^d\left(Q^u_{HH},Q^u_{FH}\right)}+\frac{F^d_{Q^u_{HH},Q^u_{FH}}\left(Q^u_{HH},Q^u_{FH}\right)}{F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)}\right] && \text{(C.11)}\\
\mu_u &= \mu_{TB}\frac{\theta-1}{\theta}P^u_{HF}+\mu_{SB}\frac{1}{\theta}\frac{1}{Q^u_{HF}}\frac{P^u_{HF}}{P^d_{HF}} && \text{(C.12)}
\end{aligned}
$$

In these derivations, note that we have used

$$
\begin{aligned}
\frac{\partial\left(\bar{A}^d_H\left(F^d\left(Q^u_{HH},Q^u_{FH}\right)\right)^{\gamma^d}F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)\right)}{\partial Q^u_{HH}} &= \bar{A}^d_H\left(F^d\left(Q^u_{HH},Q^u_{FH}\right)\right)^{\gamma^d}F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)\\
&\quad\times\left[\gamma^d\frac{F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)}{F^d\left(Q^u_{HH},Q^u_{FH}\right)}+\frac{F^d_{Q^u_{HH},Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)}{F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)}\right]
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial\left(\bar{A}^d_H\left(F^d\left(Q^u_{HH},Q^u_{FH}\right)\right)^{\gamma^d}F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)\right)}{\partial Q^u_{FH}} &= \bar{A}^d_H\left(F^d\left(Q^u_{HH},Q^u_{FH}\right)\right)^{\gamma^d}F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)\\
&\quad\times\left[\gamma^d\frac{F^d_{Q^u_{FH}}\left(Q^u_{HH},Q^u_{FH}\right)}{F^d\left(Q^u_{HH},Q^u_{FH}\right)}+\frac{F^d_{Q^u_{HH},Q^u_{FH}}\left(Q^u_{HH},Q^u_{FH}\right)}{F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)}\right].
\end{aligned}
$$

From equations (C.7), (C.8), and (C.9), we obtain:

$$
\frac{U_{Q^d_{FH}}\left(Q^d_{HH},Q^d_{FH}\right)}{U_{Q^d_{HH}}\left(Q^d_{HH},Q^d_{FH}\right)}=\frac{P^d_{FH}}{P^d_{HF}}\left(\frac{\sigma}{\sigma-1}+\frac{\mu_{SB}}{\mu_d}\frac{1}{\sigma-1}\frac{1}{Q^d_{HF}}\frac{P^u_{HF}}{P^d_{HF}}\right).
$$

Because in a competitive equilibrium with import tariffs we have

$$
\frac{U_{Q^d_{FH}}\left(Q^d_{HH},Q^d_{FH}\right)}{U_{Q^d_{HH}}\left(Q^d_{HH},Q^d_{FH}\right)}=\left(1+t^d_H\right)\frac{P^d_{FH}}{P^d_{HF}}, \tag{C.13}
$$

we can establish that

$$1+t_H^d=\frac{\sigma}{\sigma-1}+\frac{\mu_{SB}}{\mu_d}\frac{1}{\sigma-1}\frac{1}{Q_{HF}^d}\frac{P_{HF}^u}{P_{HF}^d}. \tag{C.14}$$

Also, note from equations (C.7) and (C.8), as well as (C.13), that

$$1+t_H^d=\frac{\mu_{TB}P_{HF}^d}{\mu_d}. \tag{C.15}$$

In a competitive equilibrium with import tariffs, we also have that

$$\frac{F_{Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F_{Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}=\frac{P_{HF}^u}{\left(1+t_H^u\right)P_{FH}^u}. \tag{C.16}$$

Furthermore, the last constraint in the planner problem can be written as:

$$\bar{A}_H^d\left(F^d\left(Q_{HH}^u,Q_{FH}^u\right)\right)^{\gamma^d}F_{Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)=\frac{P_{HF}^u}{P_{HF}^d}. \tag{C.17}$$

Now combine equations (C.11), (C.15), (C.16), and (C.17) to obtain

$$\frac{1+t_H^d}{1+t_H^u}=1+\gamma^d+\frac{\mu_{SB}}{\mu_d}\left[\gamma^d\frac{F_{Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F^d\left(Q_{HH}^u,Q_{FH}^u\right)}+\frac{F_{Q_{HH}^u,Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F_{Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}\right]. \tag{C.18}$$

We next work with equations (C.10) and plug in (C.11) and (C.17) to obtain

$$\mu_{TB}P_{FH}^u\frac{F_{Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F_{Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}=\mu_u+\mu_{SB}\frac{P_{HF}^u}{P_{HF}^d}\left[\frac{F_{Q_{HH}^u,Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F_{Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}-\frac{F_{Q_{HH}^u,Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F_{Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}\right]$$

And plugging μ_u from equation (C.12), we get

$$\frac{\mu_{TB}}{\mu_{SB}\frac{P_{HF}^u}{P_{HF}^d}}=\frac{\frac{1}{\theta}\frac{1}{Q_{HF}^u}+\frac{F_{Q_{HH}^u,Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F_{Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}-\frac{F_{Q_{HH}^u,Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F_{Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}}{P_{FH}^u\frac{F_{Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F_{Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}-\frac{\theta-1}{\theta}P_{HF}^u}$$

Invoking equation (C.16) we can simplify this last expression further to

$$\frac{\mu_{TB}}{\mu_{SB}}P_{HF}^d=\frac{\frac{1}{\theta}\frac{1}{Q_{HF}^u}+\frac{F_{Q_{HH}^u,Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F_{Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}-\frac{F_{Q_{HH}^u,Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F_{Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}}{\frac{1}{\left(1+t_H^u\right)}-\frac{\theta-1}{\theta}} \tag{C.19}$$

The three equations (C.14), (C.18), and (C.19) are sufficient to characterize the properties of second-best import tariffs. In particular, these equations can be reduced to

$$1+t_H^d = \frac{\sigma}{\sigma-1}+\left[\frac{1+t_H^d}{1+t_H^u}-\frac{\theta-1}{\theta}\left(1+t_H^d\right)\right]\frac{A}{C} \tag{C.20}$$

$$\frac{1+t_H^d}{1+t_H^u} = 1+\gamma^d+\left[\frac{1+t_H^d}{1+t_H^u}-\frac{\theta-1}{\theta}\left(1+t_H^d\right)\right]\frac{B}{C}, \tag{C.21}$$

where

$$
\begin{aligned}
A &= \frac{1}{\sigma-1}\frac{1}{Q_{HF}^{d}}\frac{P_{HF}^{u}}{P_{HF}^{d}} > 0;\\
B &= \gamma^{d}\frac{F_{Q_{HH}^{u}}^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)}{F^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)} + \frac{F_{Q_{HH}^{u},Q_{FH}^{u}}^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)}{F_{Q_{FH}^{u}}^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)};\\
C &= \frac{1}{\theta}\frac{1}{Q_{HF}^{u}} + \frac{F_{Q_{HH}^{u},Q_{FH}^{u}}^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)}{F_{Q_{FH}^{u}}^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)} - \frac{F_{Q_{HH}^{u},Q_{HH}^{u}}^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)}{F_{Q_{HH}^{u}}^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)}.
\end{aligned}
$$

Using

$$
F^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right) = \left(\left(Q_{HH}^{u}\right)^{\frac{\theta-1}{\theta}} + \left(Q_{FH}^{u}\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}},
$$

it is easy to verify that

$$
\begin{aligned}
B &= \gamma^{d}\frac{F_{Q_{HH}^{u}}^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)}{F^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)} + \frac{F_{Q_{HH}^{u},Q_{FH}^{u}}^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)}{F_{Q_{FH}^{u}}^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)} = \left(\gamma^{d}+\frac{1}{\theta}\right)\frac{1}{Q_{HH}^{u}}\frac{\left(Q_{HH}^{u}\right)^{\frac{\theta-1}{\theta}}}{\left(Q_{HH}^{u}\right)^{\frac{\theta-1}{\theta}} + \left(Q_{FH}^{u}\right)^{\frac{\theta-1}{\theta}}} > 0\\
C &= \frac{1}{\theta}\frac{1}{Q_{HF}^{u}} + \frac{F_{Q_{HH}^{u},Q_{FH}^{u}}^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)}{F_{Q_{FH}^{u}}^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)} - \frac{F_{Q_{HH}^{u},Q_{HH}^{u}}^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)}{F_{Q_{HH}^{u}}^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)} = \frac{1}{\theta}\left(\frac{1}{Q_{HH}^{u}}+\frac{1}{Q_{HF}^{u}}\right) > 0.
\end{aligned}
$$

Now note that, manipulating (C.20) and (C.21), we obtain

$$
\begin{aligned}
1+t_{H}^{d} &= \frac{\sigma}{\sigma-1} + \left[\frac{1+t_{H}^{d}}{1+t_{H}^{u}} - \frac{\theta-1}{\theta}\left(1+t_{H}^{d}\right)\right]\frac{A}{C}\\
\frac{1+t_{H}^{d}}{1+t_{H}^{u}} &= 1+\gamma^{d} + \left[\frac{1+t_{H}^{d}}{1+t_{H}^{u}} - \frac{\theta-1}{\theta}\left(1+t_{H}^{d}\right)\right]\frac{B}{C},
\end{aligned}
$$

Solving this system delivers

$$
1+t_{H}^{d} = \frac{\sigma}{\sigma-1}\frac{1-\frac{B}{C}+\frac{1+\gamma^{d}}{\frac{\sigma}{\sigma-1}}\frac{A}{C}}{1-\frac{B}{C}+\frac{\theta-1}{\theta}\frac{A}{C}}
$$

and

$$
\frac{1+t_{H}^{d}}{1+t_{H}^{u}} = \left(1+\gamma^{d}\right)\frac{1+\frac{\theta-1}{\theta}\frac{A}{C}-\frac{\theta-1}{\theta}\frac{\frac{\sigma}{\sigma-1}}{1+\gamma^{d}}\frac{B}{C}}{1+\frac{\theta-1}{\theta}\frac{A}{C}-\frac{B}{C}}.
$$

Noting that $1+\gamma^{d} = \frac{\sigma}{\sigma-1}$ in our isomorphism, immediately implies

$$
1+t_{H}^{d} > \frac{\sigma}{\sigma-1}
$$

and

$$
\frac{1+t_{H}^{d}}{1+t_{H}^{u}} > 1+\gamma^{d} = \frac{\sigma}{\sigma-1}.
$$

This proves Proposition 6.

B. Second-Best Import Tariffs with No Scale Economies

Given the above derivations, it is straightforward to prove Proposition 7. Simply set $\gamma^d = 0$ in the system (C.20) and (C.21), and obtain

$$1 + t_H^d = \frac{\sigma}{\sigma - 1} \frac{1 - \frac{B}{C} + \frac{\sigma-1}{\sigma}\frac{A}{C}}{1 - \frac{B}{C} + \frac{\theta-1}{\theta}\frac{A}{C}}$$

and

$$\frac{1 + t_H^d}{1 + t_H^u} = \frac{1 + \frac{\theta-1}{\theta}\frac{A}{C} - \frac{\theta-1}{\theta}\frac{\sigma}{\sigma-1}\frac{B}{C}}{1 + \frac{\theta-1}{\theta}\frac{A}{C} - \frac{B}{C}}.$$

From the second equation, is is clear that $\frac{1+t_H^d}{1+t_H^u} > 1$ if and only if $\frac{\theta-1}{\theta}\frac{\sigma}{\sigma-1} < 1$, or $\sigma > \theta$. Furthermore, when $\theta = \sigma$, we have

$$1 + t_H^d = 1 + t_H^u = \frac{\sigma}{\sigma - 1} = \frac{\theta}{\theta - 1}.$$

D Optimal Trade Policy for a Small Open Economy with Domestic Distortions: Derivations

D.1 First-Best Policies with an Upstream Production Subsidy

We begin by characterizing the solution to the program

$$\max \quad U\left(Q^d_{HH}, Q^d_{FH}\right) = \left(\left(Q^d_{HH}\right)^{\frac{\sigma-1}{\sigma}} + \left(Q^d_{FH}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

$$\begin{aligned}
s.t. \quad & L^u_H + L^d_H = L_H \\
& \hat{A}^u_H\left(L^u_H\right) L^u_H = Q^u_{HH} + Q^u_{HF} \\
& \hat{A}^d_H\left(F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)\right) F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) = Q^d_{HH} + Q^d_{HF} \\
& P^d_{FH} Q^d_{FH} + P^u_{FH} Q^u_{FH} = Q^d_{HF}(Q^d_{HF})^{-\frac{1}{\sigma}} P^d_{FF}\left(Q^d_{FF}\right)^{\frac{1}{\sigma}} + Q^u_{HF}\left(Q^u_{HF}\right)^{-\frac{1}{\theta}} P^u_{FF}\left(Q^u_{FF}\right)^{\frac{1}{\theta}},
\end{aligned}$$

where $\hat{A}^u_H$ and $\hat{A}^d_H$ are given by

$$\hat{A}^d_H = \bar{A}^d_H\left(F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)\right)^{\gamma^d}$$

and

$$\hat{A}^u_H = \bar{A}^u_H\left(L_H\right)^{\gamma^u},$$

respectively. We also note that

$$U\left(Q^d_{HH}, Q^d_{FH}\right) = \left(\left(Q^d_{HH}\right)^{\frac{\sigma-1}{\sigma}} + \left(Q^d_{FH}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

and that

$$F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) = \left(L^d_i\right)^{\alpha}\left(\left(Q^u_{HH}\right)^{\frac{\theta-1}{\theta}} + \left(Q^u_{FH}\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}},$$

but this will prove immaterial for the derivations below.

We first write the Lagrangian of this problem

$$\begin{aligned}
& U\left(Q^d_{HH}, Q^d_{FH}\right) + \mu_L\left[L_H - L^u_H - L^d_H\right] + \mu_u\left[\bar{A}^u_H\left(L_H\right)^{1+\gamma^u} - Q^u_{HH} - Q^u_{HF}\right] \\
& +\mu_d\left[\bar{A}^d_H\left(F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)\right)^{1+\gamma^d} - Q^d_{HH} - Q^d_{HF}\right] \\
& +\mu_{TB}\left[Q^d_{HF}(Q^d_{HF})^{-\frac{1}{\sigma}} P^d_{FF}\left(Q^d_{FF}\right)^{\frac{1}{\sigma}} + Q^u_{HF}\left(Q^u_{HF}\right)^{-\frac{1}{\theta}} P^u_{FF}\left(Q^u_{FF}\right)^{\frac{1}{\theta}} - P^d_{FH} Q^d_{FH} - P^u_{FH} Q^u_{FH}\right].
\end{aligned}$$

The first order conditions associated with the choices of Q^d_{HH}, Q^d_{FH}, Q^d_{HF}, Q^u_{HH}, Q^u_{FH}, Q^u_{HF}, L^d_H, and L^u_H

are as follows:

$$U_{Q^d_{HH}}\left(Q^d_{HH},Q^d_{FH}\right) = \mu_d \quad \text{(D.1)}$$
$$U_{Q^d_{FH}}\left(Q^d_{HH},Q^d_{FH}\right) = \mu_{TB}P^d_{FH} \quad \text{(D.2)}$$
$$\mu_d = \mu_{TB}\frac{\sigma-1}{\sigma}P^d_{HF} \quad \text{(D.3)}$$
$$\mu_u = \mu_d\left(1+\gamma^d\right)\bar{A}^d_H\left(F^d\left(L^d_H,Q^u_{HH},Q^u_{FH}\right)\right)^{\gamma^d}F^d_{Q^u_{HH}}\left(L^d_H,Q^u_{HH},Q^u_{FH}\right) \quad \text{(D.4)}$$
$$\mu_{TB}P^u_{FH} = \mu_d\left(1+\gamma^d\right)\bar{A}^d_H\left(F^d\left(L^d_H,Q^u_{HH},Q^u_{FH}\right)\right)^{\gamma^d}F^d_{Q^u_{FH}}\left(L^d_H,Q^u_{HH},Q^u_{FH}\right) \quad \text{(D.5)}$$
$$\mu_u = \mu_{TB}\frac{\theta-1}{\theta}P^u_{HF} \quad \text{(D.6)}$$
$$\mu_L = \mu_u\left(1+\gamma^u\right)\bar{A}^u_H\left(L_H\right)^{\gamma^u} \quad \text{(D.7)}$$
$$\mu_L = \mu_d\left(1+\gamma^d\right)\bar{A}^d_H\left(F^d\left(L^d_H,Q^u_{HH},Q^u_{FH}\right)\right)^{\gamma^d}F^d_{L^d_H}\left(L^d_H,Q^u_{HH},Q^u_{FH}\right) \quad \text{(D.8)}$$

Dividing equation (D.1) by equation (D.2), and plugging in (D.3), we obtain obtain:

$$\frac{U_{Q^d_{HH}}\left(Q^d_{HH},Q^d_{FH}\right)}{U_{Q^d_{FH}}\left(Q^d_{HH},Q^d_{FH}\right)} = \frac{\frac{\sigma-1}{\sigma}P^d_{HF}}{P^d_{FH}},$$

which corresponds to the first optimality condition (23) in the main text.

Next, we divide equation (D.4) by equation (D.5), and plugging in (D.6), delivers

$$\frac{F^d_{Q^u_{HH}}\left(L^d_H,Q^u_{HH},Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(L^d_H,Q^u_{HH},Q^u_{FH}\right)} = \frac{\frac{\theta-1}{\theta}P^u_{HF}}{P^u_{FH}},$$

which corresponds to the second optimality condition (24) in the main text.

Next, combining equation (D.4) with the ratio of equations (D.3) and (D.6) produces

$$\left(1+\gamma^d\right)\bar{A}^d_H\left(F^d\left(L^d_H,Q^u_{HH},Q^u_{FH}\right)\right)^{\gamma^d}F^d_{Q^u_{HH}}\left(L^d_H,Q^u_{HH},Q^u_{FH}\right) = \frac{\frac{\theta-1}{\theta}}{\frac{\sigma-1}{\sigma}}\frac{P^u_{HF}}{P^d_{HF}},$$

which corresponds to the third optimality condition (25) in the main text.

Finally, from equations (D.7) by equation (D.8), and plugging in (D.4), we obtain

$$F^d_{L^d_H}\left(L^d_H,Q^u_{HH},Q^u_{FH}\right) = \left(1+\gamma^u\right)\bar{A}^u_H\left(L_H\right)^{\gamma^u}F^d_{Q^u_{HH}}\left(L^d_H,Q^u_{HH},Q^u_{FH}\right),$$

which corresponds to equation (31) in the main text, after noting that $\bar{A}^u_H\left(L_H\right)^{\gamma^u} = \hat{A}^u_H$.

D.2 First-Best Policies with Alternative Instruments

In this Appendix, we explore the structure of first-best policies when the set of available instruments includes instruments other than domestic upstream production subsidies and trade taxes.

D.2.1 Discriminatory Domestic Subsidies

Consider first the case in which the Home government has access to discriminatory domestic subsidies s^d_{HH} and s^u_{HH} that apply only to purchases of final goods and of intermediate inputs involving only Home residents.

The inclusion of these instruments alters the decentralized market equilibrium conditions (18), (19), (20), and (21) as follows:

$$\begin{aligned}
\frac{U_{Q^d_{HH}}\left(Q^d_{HH}, Q^d_{FH}\right)}{U_{Q^d_{FH}}\left(Q^d_{HH}, Q^d_{FH}\right)} &= \left(1 - s^d_{HH}\right)\frac{\left(1 - v^d_H\right)}{\left(1 + t^d_H\right)}\frac{P^d_{HF}}{P^d_{FH}}; \\
\frac{F^d_{Q^u_{HH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)} &= \left(1 - s^u_{HH}\right)\frac{\left(1 - v^u_H\right)}{\left(1 + t^u_H\right)}\frac{P^u_{HF}}{P^u_{FH}}; \\
\hat{A}^d_H F^d_{Q^u_{HH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) &= \left(1 - s^u_{HH}\right)\frac{\left(1 - v^u_H\right)}{\left(1 - v^d_H\right)}\frac{P^u_{HF}}{P^d_{HF}} \\
F^d_{L^d_H}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) &= \frac{1}{1 - s^u_{HH}}\hat{A}^u\left(L^u_H\right) F^d_{Q^u_{HH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)
\end{aligned}$$

Comparing these equations to those characterizing the optimal allocations, that is equations (23), (24), (25), and (31), it is clear that the first best can be achieved by setting

$$\begin{aligned}
\left(1 + t^d_H\right)\left(1 - s^d_{HH}\right) &= \left(1 + \gamma^d\right)\left(1 + \bar{T}\right); \\
1 + t^u_H &= 1 + \bar{T}; \\
1 - v^d_H &= \frac{\sigma - 1}{\sigma}\left(1 + \gamma^d\right)\left(1 + \bar{T}\right); \\
\left(1 - s^u_{HH}\right)\left(1 - v^u_H\right) &= \frac{\theta - 1}{\theta}\left(1 + \bar{T}\right); \\
\frac{1}{1 - s^u_{HH}} &= \frac{\theta - 1}{\theta}.
\end{aligned}$$

These equations illustrate that a downstream discriminatory subsidy is a perfect substitute for the downstream import tariff (only the product $\left(1 + t^d_H\right)\left(1 - s^d_{HH}\right)$ matters), while an upstream discriminatory subsidy is *no longer* a perfect substitute for the upstream export tax, as it was in the case in which $\alpha = 0$. More specifically, because the first-best calls for $s^u_{HH} = 1/\theta$, we must necessarily have

$$1 - v^u_H = 1 + t^u_H = 1 + \bar{T}.$$

A straightforward implication of this result is that, whenever $1 + \gamma^d = \sigma/\left(\sigma - 1\right)$, as imposed by our isomorphism, the first best can be attained by setting $\left(1 + t^d_H\right)\left(1 - s^d_{HH}\right) = \sigma/\left(\sigma - 1\right)$, $1 - s^u_{HH} = \left(\theta - 1\right)/\theta$, and $v^u_H = v^d_H = t^u_H = 0$. Thus, the first best can be achieved with only two discriminatory subsidies, or with a combination of an upstream subsidy and a downstream import tariff. When only domestic instruments are used, we necessarily have $s^d_{HH} = 1/\sigma$ and $s^u_{HH} = 1/\theta$. Whether or not the resulting first-best policies entail tariff escalation depends on the level of the downstream domestic subsidy, since $\left(1 + t^d_H\right)/\left(1 + t^u_H\right) = \left(1 + \gamma^d\right)/\left(1 - s^d_{HH}\right)$.

D.2.2 Production Subsidies

We next consider the case in which the Home government uses a nondiscriminatory downstream production subsidy s^d_H in addition to a nondiscriminatory upstream production subsidy, as in our baseline implementation. The inclusion of this instrument does not affect the market equilibrium condition (21), while it alters the decentralized market equilibrium conditions (18), (19), (20) in a manner analogous to the case $\alpha = 0$, that is:

$$\begin{aligned}
\frac{U_{Q^d_{HH}}\left(Q^d_{HH}, Q^d_{FH}\right)}{U_{Q^d_{FH}}\left(Q^d_{HH}, Q^d_{FH}\right)} &= \frac{\left(1-v^d_H\right)}{\left(1+t^d_H\right)}\frac{P^d_{HF}}{P^d_{FH}};\\
\frac{F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(Q^u_{HH}, Q^u_{FH}\right)} &= \frac{\left(1-v^u_H\right)}{\left(1+t^u_H\right)}\frac{P^u_{HF}}{P^u_{FH}};\\
\hat{A}^d_H F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right) &= \left(1-s^d_H\right)\frac{\left(1-v^u_H\right)}{\left(1-v^d_H\right)}\frac{P^u_{HF}}{P^d_{HF}}.
\end{aligned}$$

Comparing these equations to those characterizing the optimal allocations, that is equations (23), (24), and (25), it is clear that the first best can be achieved by setting

$$\begin{aligned}
1+t^d_H &= \left(1-s^d_H\right)\left(1+\gamma^d\right)\left(1+\bar{T}\right);\\
1+t^u_H &= 1+\bar{T};\\
1-v^d_H &= \frac{\sigma-1}{\sigma}\left(1+\gamma^d\right)\left(1-s^d_H\right)\left(1+\bar{T}\right);\\
1-v^u_H &= \frac{\theta-1}{\theta}\left(1+\bar{T}\right).
\end{aligned}$$

Notice that, as long as $s^d_H > 0$, the set of first-best policies will entail this subsidy and at least two additional trade instruments. For instance, when setting $s^d_H = \gamma^d/\left(1+\gamma^d\right)$, the first best can be achieved with this production subsidy, the upstream production subsidy at a level $s^u_H = \gamma^u/\left(1+\gamma^u\right)$ and two export taxes ($1-v^d_H = \left(\sigma-1\right)/\sigma$ and $1-v^u_H = \left(\theta-1\right)/\theta$), while setting all import tariffs to zero. Alternatively, when setting $s^d_H = \left(1+\gamma^u\right)/\gamma^u = \left(\theta-1\right)/\theta$, the first best can be achieved with this production subsidy and two import tariffs ($t^d_H = 1/\left(\sigma-1\right)$ and $t^u_H = 1/\left(\theta-1\right)$).

Regardless of the actual implementation, the tariff escalation wedge is given by:

$$\frac{1+t^d_H}{1+t^u_H} = \left(1-s^d_H\right)\left(1+\gamma^d\right).$$

When only upstream production subsidies and trade taxes are used, the first-best policies continues to feature a tariff escalation wedge equal to $1+\gamma^d = \sigma/\left(\sigma-1\right)$.

D.2.3 Consumption Subsidies

We finally consider the case of consumption subsidies s^d_H and s^u_H that apply to Home consumption of final goods and of intermediate inputs, regardless of where those goods are purchased (domestically or imported). The inclusion of these instruments does not affect the market equilibrium condition (21), as long as s^u_H is set at $s^u_H = 1/\theta$. Furthermore, the decentralized market equilibrium conditions (18), (19) and (20) become:

$$\begin{aligned}
\frac{U_{Q^d_{HH}}\left(Q^d_{HH}, Q^d_{FH}\right)}{U_{Q^d_{FH}}\left(Q^d_{HH}, Q^d_{FH}\right)} &= \frac{\left(1-v^d_H\right)}{\left(1+t^d_H\right)}\frac{P^d_{HF}}{P^d_{FH}};\\
\frac{F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(Q^u_{HH}, Q^u_{FH}\right)} &= \frac{\left(1-v^u_H\right)}{\left(1+t^u_H\right)}\frac{P^u_{HF}}{P^u_{FH}};\\
\hat{A}^d_H F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right) &= \left(1-s^u_H\right)\frac{\left(1-v^u_H\right)}{\left(1-v^d_H\right)}\frac{P^u_{HF}}{P^d_{HF}}.
\end{aligned}$$

These equations are completely analogous to those applying to the case of prouduction subsidies, with s_H^u replacing s_H^d, but note that we now necessarily have $s_H^u = 1/\theta$. As a result, replacing $1+\gamma^d = \sigma/(\sigma-1)$, we obtain

$$\begin{aligned}
1+t_H^d &= \frac{\theta-1}{\theta}\left(1+\gamma^d\right)\left(1+\bar{T}\right); \\
1+t_H^u &= 1+\bar{T}; \\
1-v_H^d &= \frac{\theta-1}{\theta}\left(1+\bar{T}\right); \\
1-v_H^u &= \frac{\theta-1}{\theta}\left(1+\bar{T}\right).
\end{aligned}$$

In such a case, it is clear that the relative size of $1+t_H^d$ and $1+t_H^u$ depends on $\frac{\theta-1}{\theta}\left(1+\gamma^d\right) = \frac{\theta-1}{\theta}\frac{\sigma}{\sigma-1}$, and thus on the relative size of σ and θ.

D.3 Second-Best Policies

In this Appendix we characterize the second-best import tariffs for the general case $\alpha \geq 0$ when the government only has access to import tariffs upstream and downstream. We first derive the key equations characterizing tariff levels and tariff escalation, and we later explore special cases.

A. Second-Best Import Tariffs with Scale Economies: Key Equations

The second-best optimal allocation will seek to solve the same problem laid out in section 4.1 expanded to include the additional constraints:

$$\hat{A}_H^d F_{Q_{HH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right) = \frac{P_{HF}^d}{P_{HF}^u} = \frac{\left(Q_{HF}^u\right)^{-\frac{1}{\theta}} P_{FF}^u \left(Q_{FF}^u\right)^{\frac{1}{\theta}}}{\left(Q_{HF}^d\right)^{-\frac{1}{\sigma}} P_{FF}^d \left(Q_{FF}^d\right)^{\frac{1}{\sigma}}}$$

and

$$F_{L_H^d}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) = \hat{A}^u\left(L_H^u\right) F_{Q_{HH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right).$$

Second-Best Planner Problem and First-Order Conditions More specifically, the planner sets $\left\{L_H^u, L_H^d, Q_{HH}^d, Q_{FH}^d, Q_{HF}^d, Q_{HH}^u, Q_{FH}^u, Q_{HF}^u\right\}$ to

$$\max \quad U\left(Q_{HH}^d, Q_{FH}^d\right) = \left(\left(Q_{HH}^d\right)^{\frac{\sigma-1}{\sigma}} + \left(Q_{FH}^d\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

$$\begin{aligned}
s.t. \quad & L_H^u + L_H^d = L_H \\
& \hat{A}_H^u\left(L_H^u\right) L_H^u = Q_{HH}^u + Q_{HF}^u \\
& \hat{A}_H^d\left(F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)\right) F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) = Q_{HH}^d + Q_{HF}^d \\
& P_{FH}^d Q_{FH}^d + P_{FH}^u Q_{FH}^u = Q_{HF}^d (Q_{HF}^d)^{-\frac{1}{\sigma}} P_{FF}^d \left(Q_{FF}^d\right)^{\frac{1}{\sigma}} + Q_{HF}^u \left(Q_{HF}^u\right)^{-\frac{1}{\theta}} P_{FF}^u \left(Q_{FF}^u\right)^{\frac{1}{\theta}} \\
& \hat{A}_H^d F_{Q_{HH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right) = \frac{\left(Q_{HF}^u\right)^{-\frac{1}{\theta}} P_{FF}^u \left(Q_{FF}^u\right)^{\frac{1}{\theta}}}{\left(Q_{HF}^d\right)^{-\frac{1}{\sigma}} P_{FF}^d \left(Q_{FF}^d\right)^{\frac{1}{\sigma}}} \\
& F_{L_H^d}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) = \hat{A}^u\left(L_H^u\right) F_{Q_{HH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)
\end{aligned}$$

where $\hat{A}_H^u$ and $\hat{A}_H^d$ are given by

$$\hat{A}_H^d = \bar{A}_H^d \left(F^d \left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)\right)^{\gamma^d}$$

and

$$\hat{A}_H^u = \bar{A}_H^u \left(L_H^u\right)^{\gamma^u},$$

respectively.

We first write the Lagrangian of this problem

$$\begin{aligned}
&U\left(Q_{HH}^d, Q_{FH}^d\right) + \mu_u \left[\bar{A}_H^u \left(L_H^u\right)^{1+\gamma^u} - Q_{HH}^u - Q_{HF}^u\right] + \mu_d \left[\bar{A}_H^d \left(F^d \left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)\right)^{1+\gamma^d} - Q_{HH}^d - Q_{HF}^d\right] \\
&+\mu_{TB} \left[Q_{HF}^d (Q_{HF}^d)^{-\frac{1}{\sigma}} P_{FF}^d \left(Q_{FF}^d\right)^{\frac{1}{\sigma}} + Q_{HF}^u \left(Q_{HF}^u\right)^{-\frac{1}{\theta}} P_{FF}^u \left(Q_{FF}^u\right)^{\frac{1}{\theta}} - P_{FH}^d Q_{FH}^d - P_{FH}^u Q_{FH}^u\right] \\
&+\mu_{SB} \left[\hat{A}_H^d F_{Q_{HH}^u}^d \left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) - \frac{\left(Q_{HF}^u\right)^{-\frac{1}{\theta}} P_{FF}^u \left(Q_{FF}^u\right)^{\frac{1}{\theta}}}{(Q_{HF}^d)^{-\frac{1}{\sigma}} P_{FF}^d \left(Q_{FF}^d\right)^{\frac{1}{\sigma}}}\right] \\
&+\mu_{LC} \left[\frac{F_{L_H^d}^d \left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)}{F_{Q_{HH}^u}^d \left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)} - \hat{A}^u \left(L_H^u\right)\right]
\end{aligned}$$

The first order conditions associated with the choices of Q_{HH}^d, Q_{FH}^d, Q_{HF}^d, Q_{HH}^u, Q_{FH}^u, Q_{HF}^u, L_H^d, and L_H^u are as follows:

$$U_{Q^d_{HH}}\left(Q^d_{HH},Q^d_{FH}\right) = \mu_d \tag{D.9}$$

$$U_{Q^d_{FH}}\left(Q^d_{HH},Q^d_{FH}\right) = \mu_{TB}P^d_{FH} \tag{D.10}$$

$$\mu_d = \mu_{TB}\frac{\sigma-1}{\sigma}P^d_{HF} - \mu_{SB}\frac{1}{\sigma}\frac{1}{Q^d_{HF}}\frac{P^u_{HF}}{P^d_{HF}} \tag{D.11}$$

$$\begin{aligned}\mu_u =\ & \mu_d\left(1+\gamma^d\right)\bar{A}^d_H\left(F^d(\cdot)\right)^{\gamma^d}F^d_{Q^u_{HH}}(\cdot) \\ & +\mu_{SB}\bar{A}^d_H\left(F^d(\cdot)\right)^{\gamma^d}F^d_{Q^u_{HH}}(\cdot) \\ & \times\left[\gamma^d\frac{F^d_{Q^u_{HH}}(\cdot)}{F^d(\cdot)}+\frac{F^d_{Q^u_{HH},Q^u_{HH}}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)}\right] \\ & +\mu_{LC}\left[\frac{F^d_{L^d_H,Q^u_{HH}}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)}-\frac{F^d_{L^d_H}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)}\frac{F^d_{Q^u_{HH},Q^u_{HH}}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)}\right]\end{aligned} \tag{D.12}$$

$$\begin{aligned}\mu_{TB}P^u_{FH} =\ & \mu_d\left(1+\gamma^d\right)\bar{A}^d_H\left(F^d(\cdot)\right)^{\gamma^d}F^d_{Q^u_{FH}}(\cdot) \\ & +\mu_{SB}\bar{A}^d_H\left(F^d(\cdot)\right)^{\gamma^d}F^d_{Q^u_{HH}}(\cdot)\times\left[\gamma^d\frac{F^d_{Q^u_{FH}}(\cdot)}{F^d(\cdot)}+\frac{F^d_{Q^u_{HH},Q^u_{FH}}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)}\right] \\ & +\mu_{LC}\left[\frac{F^d_{L^d_H,Q^u_{FH}}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)}-\frac{F^d_{L^d_H}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)}\frac{F^d_{Q^u_{HH},Q^u_{FH}}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)}\right]\end{aligned} \tag{D.13}$$

$$\mu_u = \mu_{TB}\frac{\theta-1}{\theta}P^u_{HF} + \mu_{SB}\frac{1}{\theta}\frac{1}{Q^u_{HF}}\frac{P^u_{HF}}{P^d_{HF}} \tag{D.14}$$

$$\mu_L = \mu_u\left(1+\gamma^u\right)\bar{A}^u_H\left(L^u_H\right)^{\gamma^u} - \mu_{LC}\gamma^u\bar{A}^u_H\left(L^u_H\right)^{\gamma^u-1} \tag{D.15}$$

$$\begin{aligned}\mu_L =\ & \mu_d\left(1+\gamma^d\right)\bar{A}^d_H\left(F^d(\cdot)\right)^{\gamma^d}F^d_{L^d_H}(\cdot) \\ & +\mu_{SB}\hat{A}^d_H\left[\gamma^d\frac{F^d_{L^d_H}(\cdot)}{F^d(\cdot)}F^d_{Q^u_{HH}}(\cdot)+F^d_{Q^u_{HH},L^d_H}(\cdot)\right] \\ & +\mu_{LC}\left[\frac{F^d_{L^d_H,L^d_H}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)}-\frac{F^d_{L^d_H}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)}\frac{F^d_{Q^u_{HH},L^d_H}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)}\right]\end{aligned} \tag{D.16}$$

In these derivations, note that we have used

$$\begin{aligned}\frac{\partial\left(\bar{A}^d_H\left(F^d(\cdot)\right)^{\gamma^d}F^d_{Q^u_{HH}}(\cdot)\right)}{\partial Q^u_{HH}} =\ & \bar{A}^d_H\left(F^d(\cdot)\right)^{\gamma^d}F^d_{Q^u_{HH}}(\cdot) \\ & \times\left[\gamma^d\frac{F^d_{Q^u_{HH}}(\cdot)}{F^d(\cdot)}+\frac{F^d_{Q^u_{HH},Q^u_{HH}}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)}\right]\end{aligned}$$

and

$$\begin{aligned}\frac{\partial\left(\bar{A}^d_H\left(F^d(\cdot)\right)^{\gamma^d}F^d_{Q^u_{HH}}(\cdot)\right)}{\partial Q^u_{FH}} =\ & \bar{A}^d_H\left(F^d(\cdot)\right)^{\gamma^d}F^d_{Q^u_{HH}}(\cdot) \\ & \times\left[\gamma^d\frac{F^d_{Q^u_{FH}}(\cdot)}{F(\cdot)}+\frac{F^d_{Q^u_{HH},Q^u_{FH}}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)}\right].\end{aligned}$$

Useful Expressions with Our Functional Forms Remember that technology is given by

$$F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) = \left(L^d_H\right)^{\alpha}\left(\left(Q^u_{HH}\right)^{\frac{\theta-1}{\theta}} + \left(Q^u_{FH}\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}},$$

and define

$$\pi^u_{HH} \equiv \frac{\left(Q^u_{HH}\right)^{\frac{\theta-1}{\theta}}}{\left(Q^u_{HH}\right)^{\frac{\theta-1}{\theta}} + \left(Q^u_{FH}\right)^{\frac{\theta-1}{\theta}}}$$

and

$$X^d_H \equiv \bar{A}^d_H\left(F^d\left(Q^u_{HH}, Q^u_{FH}\right)\right)^{1+\gamma^d}.$$

We next note that:

$$\begin{aligned}
F^d_{L^d_H}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) &= \alpha\frac{F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)}{L^d_H} \\
F^d_{Q^u_{HH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) &= (1-\alpha)\frac{F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)}{Q^u_{HH}}\pi^u_{HH} \\
F^d_{Q^u_{FH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) &= (1-\alpha)\frac{F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)}{Q^u_{FH}}\left(1-\pi^u_{HH}\right) \\
F^d_{L^d_H,L^d_H}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) &= \alpha(\alpha-1)\frac{F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)}{L^d_H L^d_H} \\
F^d_{Q^u_{HH},Q^u_{HH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) &= (1-\alpha)\frac{F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)}{Q^u_{HH}}\frac{\pi^u_{HH}}{Q^u_{HH}}\left[-\alpha\pi^u_{HH} - \frac{1}{\theta}\left(1-\pi^u_{HH}\right)\right] \\
F^d_{Q^u_{HH},Q^u_{FH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) &= (1-\alpha)F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)\frac{1-\pi^u_{HH}}{Q^u_{FH}}\frac{\pi^u_{HH}}{Q^u_{HH}}\left(\frac{1}{\theta}-\alpha\right) \\
F^d_{L^d_H,Q^u_{HH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) &= \alpha(1-\alpha)\frac{F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)}{L^d_H}\frac{\pi^u_{HH}}{Q^u_{HH}} \\
F^d_{L^d_H,Q^u_{FH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) &= \alpha(1-\alpha)\frac{F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)}{L^d_H}\frac{1-\pi^u_{HH}}{Q^u_{FH}}
\end{aligned}$$

First-Order Conditions with Functional Forms We can now plug some of the above expressions into our first-order conditions

$$\begin{aligned}
U_{Q^d_{HH}}\left(Q^d_{HH}, Q^d_{FH}\right) &= \mu_d \quad (D.17)\\
U_{Q^d_{FH}}\left(Q^d_{HH}, Q^d_{FH}\right) &= \mu_{TB} P^d_{FH} \quad (D.18)\\
\mu_d &= \mu_{TB}\frac{\sigma-1}{\sigma}P^d_{HF} - \mu_{SB}\frac{1}{\sigma}\frac{1}{Q^d_{HF}}\frac{P^u_{HF}}{P^d_{HF}} \quad (D.19)\\
\mu_u &= \mu_d\left(1+\gamma^d\right)(1-\alpha)X^d_H\frac{\pi^u_{HH}}{Q^u_{HH}} + \mu_{SB}(1-\alpha)\frac{X^d_H}{Q^u_{HH}}\frac{\pi^u_{HH}}{Q^u_{HH}}\left[\gamma^d(1-\alpha)\pi^u_{HH} - \alpha\pi^u_{HH} - \frac{1}{\theta}\pi^u_{FH}\right]\\
&\quad +\mu_{LC}\frac{1}{L^d_H}\frac{\alpha}{(1-\alpha)}\left[1+\frac{1}{\theta}\frac{1-\pi^u_{HH}}{\pi^u_{HH}}\right] \quad (D.20)\\
\mu_{TB}P^u_{FH} &= \mu_d\left(1+\gamma^d\right)(1-\alpha)X^d_H\frac{1-\pi^u_{HH}}{Q^u_{FH}} + \mu_{SB}(1-\alpha)X^d_H\frac{\pi^u_{HH}}{Q^u_{HH}}\frac{1-\pi^u_{HH}}{Q^u_{FH}}\left[\gamma^d(1-\alpha) + \left(\frac{1}{\theta}-\alpha\right)\right]\\
&\quad +\mu_{LC}\frac{\alpha}{1-\alpha}\frac{\theta-1}{\theta}\frac{1}{L^d_H}\frac{1-\pi^u_{HH}}{Q^u_{FH}}\frac{Q^u_{HH}}{\pi^u_{HH}} \quad (D.21)\\
\mu_u &= \mu_{TB}\frac{\theta-1}{\theta}P^u_{HF} + \mu_{SB}\frac{1}{\theta}\frac{1}{Q^u_{HF}}\frac{P^u_{HF}}{P^d_{HF}} \quad (D.22)\\
\mu_L &= \mu_u\left(1+\gamma^u\right)\bar{A}^u_H\left(L^u_H\right)^{\gamma^u} - \mu_{LC}\gamma^u\bar{A}^u_H\left(L^u_H\right)^{\gamma^u-1} \quad (D.23)\\
\mu_L &= \mu_d\left(1+\gamma^d\right)\frac{\alpha X^d}{L^d_H} + \mu_{SB}\left(1+\gamma^d\right)\frac{\alpha(1-\alpha)X^d_H}{L^d_H}\frac{\pi^u_{HH}}{Q^u_{HH}} - \mu_{LC}\frac{\alpha}{1-\alpha}\frac{1}{L^d_H L^d_H}\frac{Q^u_{HH}}{\pi^u_{HH}} \quad (D.24)
\end{aligned}$$

Manipulating the First-Order Conditions From equations (D.17), (D.18), and (D.19), we obtain:

$$\frac{U_{Q^d_{FH}}\left(Q^d_{HH}, Q^d_{FH}\right)}{U_{Q^d_{HH}}\left(Q^d_{HH}, Q^d_{FH}\right)} = \frac{P^d_{FH}}{P^d_{HF}}\left(\frac{\sigma}{\sigma-1} + \frac{\mu_{SB}}{\mu_d}\frac{1}{\sigma-1}\frac{1}{Q^d_{FH}}\frac{P^u_{HF}}{P^d_{HF}}\right).$$

Because in a competitive equilibrium with tariffs we have

$$\frac{U_{Q^d_{FH}}\left(Q^d_{HH}, Q^d_{FH}\right)}{U_{Q^d_{HH}}\left(Q^d_{HH}, Q^d_{FH}\right)} = \left(1+\tau^d_H\right)\frac{P^d_{FH}}{P^d_{HF}}, \quad (D.25)$$

we can establish that

$$1+\tau^d_H = \frac{\sigma}{\sigma-1} + \frac{\mu_{SB}}{\mu_d}\frac{1}{\sigma-1}\frac{1}{Q^d_{FH}}\frac{P^u_{HF}}{P^d_{HF}}. \quad (D.26)$$

Also, note from equations (D.17), (D.18), and (D.25) that we have

$$1+\tau^d_H = \frac{\mu_{TB}P^d_{HF}}{\mu_d}, \quad (D.27)$$

and in a competitive equilibrium with import tariffs (but no export taxes)

$$\frac{F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(Q^u_{HH}, Q^u_{FH}\right)} = \frac{P^u_{HF}}{\left(1+t^u_H\right)P^u_{FH}}. \quad (D.28)$$

Furthermore, the penultimate constraint in the initial optimization can be written as:

$$\frac{X^d_H}{F^d\left(Q^u_{HH}, Q^u_{FH}\right)}F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right) = \frac{P^u_{HF}}{P^d_{HF}}, \quad (D.29)$$

and the last one as

$$\frac{\alpha}{1-\alpha}\frac{Q^u_{HH}}{\pi^u_{HH}L^d_H}=\frac{X^u_H}{L^u_H}. \tag{D.30}$$

Now combine equations (D.21), (D.27), (D.28), (D.29), and (D.30)

$$\frac{1+t^d_H}{1+t^u_H}=1+\gamma^d+\frac{\mu_{SB}}{\mu_d}\frac{\pi^u_{HH}}{Q^u_{HH}}\left[\gamma^d\left(1-\alpha\right)+\left(\frac{1}{\theta}-\alpha\right)\right]+\frac{\mu_{LC}}{\mu_c}\frac{\alpha}{1-\alpha}\frac{\theta-1}{\theta}\frac{1}{L^d_H}\frac{P^d_{HF}}{P^u_{HF}}. \tag{D.31}$$

We next plug equation (D.21) into equation (D.20) to obtain

$$\mu_{TB}P^u_{FH}\frac{\pi^u_{HH}}{Q^u_{HH}}\frac{Q^u_{FH}}{1-\pi^u_{HH}}=\mu_u+\mu_{SB}\left(1-\alpha\right)\frac{X^d_H}{Q^u_{HH}}\frac{\pi^u_{HH}}{Q^u_{HH}}\frac{1}{\theta}-\mu_{LC}\frac{1}{L^d_H}\frac{\alpha}{1-\alpha}\frac{1}{\theta}\frac{1}{\pi^u_{HH}}$$

Next, plugging μ_u from equation (D.22), and invoking equation (D.28) and (D.29), we obtain

$$\mu_{TB}P^u_{HF}\left[\frac{1}{1+t^u_H}-\frac{\theta-1}{\theta}\right]=\mu_{SB}\frac{1}{\theta}\left(1-\alpha\right)X^d_H\frac{\pi^u_{HH}}{Q^u_{HH}}\left[\frac{1}{Q^u_{HF}}+\frac{1}{Q^u_{HH}}\right]-\mu_{LC}\frac{1}{L^d_H}\frac{\alpha}{\left(1-\alpha\right)}\frac{1}{\theta}\frac{1}{\pi^u_{HH}}$$

Next, invoking equation (D.28) and (D.29), we can simplify this to

$$\mu_{TB}P^u_{HF}\left[\frac{1}{1+t^u_H}-\frac{\theta-1}{\theta}\right]=\mu_{SB}\frac{1}{\theta}X^d_H\left(1-\alpha\right)\frac{\pi^u_{HH}}{Q^u_{HH}}\left[\frac{1}{Q^u_{HF}}+\frac{1}{Q^u_{HH}}\right]-\mu_{LC}\frac{\alpha}{1-\alpha}\frac{1}{L^d_H}\frac{1}{\theta}\frac{1}{\pi^u_{HH}}.$$

And, plugging in (D.27). this delivers

$$\frac{1+\tau^d_H}{1+t^u_H}-\frac{\theta-1}{\theta}\left(1+\tau^d_H\right)=\frac{\mu_{SB}}{\mu_d}\frac{1}{\theta}\left[\frac{1}{Q^u_{HF}}+\frac{1}{Q^u_{HH}}\right]-\frac{\mu_{LC}}{\mu_d}\frac{\alpha}{1-\alpha}\frac{1}{L^d_H}\frac{1}{\theta}\frac{1}{\pi^u_{HH}}\frac{P^d_{HF}}{P^u_{HF}}. \tag{D.32}$$

We finally seek to solve for μ_{LC} as a function of μ_{SB}. We begin with equation (D.15) and (D.24)

$$\frac{\mu_u}{\mu_d}\bar{A}^u_H\left(L^u_H\right)^{\gamma^u}=\frac{\left(1+\gamma^d\right)}{\left(1+\gamma^u\right)}\frac{\alpha X^d}{L^d_H}+\frac{\mu_{SB}}{\mu_d}\frac{\left(1+\gamma^d\right)}{\left(1+\gamma^u\right)}\frac{\alpha\left(1-\alpha\right)X^d_H}{L^d_H}\frac{\pi^u_{HH}}{Q^u_{HH}}+\frac{\mu_{LC}}{\mu_d}\frac{1}{1+\gamma^u}\left[\gamma^u\bar{A}^u_H\left(L^u_H\right)^{\gamma^u-1}-\frac{\alpha}{1-\alpha}\frac{1}{L^d_HL^d_H}\frac{Q^u_{HH}}{\pi^u_{HH}}\right].$$

Next, plug equation (D.12) and using (D.30), we obtain

$$\begin{aligned}&\frac{\left(1+\gamma^d\right)\gamma^u}{\left(1+\gamma^u\right)}X^d_H+\frac{\mu_{SB}}{\mu_d}\left(1-\alpha\right)\frac{X^d_H\pi^u_{HH}}{Q^u_{HH}}\left[\frac{\left(1+\gamma^d\right)}{\left(1+\gamma^u\right)}\gamma^u-\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{\left(1-\pi^u_{HH}\right)}{\pi^u_{HH}}\right)\right]\\ =\;&\frac{\mu_{LC}}{\mu_d}\bar{A}^u_H\left(L^u_H\right)^{\gamma^u}\left[\frac{1}{\alpha}\frac{1}{1+\gamma^u}\left(\frac{\gamma^uL^d_H}{L^u_H}-1\right)-\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{1-\pi^u_{HH}}{\pi^u_{HH}}\right)\right],\end{aligned}$$

which we can express as

$$\frac{\mu_{LC}}{\mu_d}=\frac{\frac{\left(1+\gamma^d\right)\gamma^u}{\left(1+\gamma^u\right)}X^d_H+\frac{\mu_{SB}}{\mu_d}\left(1-\alpha\right)\frac{X^d_H\pi^u_{HH}}{Q^u_{HH}}\left[\frac{\left(1+\gamma^d\right)}{\left(1+\gamma^u\right)}\gamma^u-\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{\left(1-\pi^u_{HH}\right)}{\pi^u_{HH}}\right)\right]}{\bar{A}^u_H\left(L^u_H\right)^{\gamma^u}\left[\frac{1}{\alpha}\frac{1}{1+\gamma^u}\left(\frac{\gamma^uL^d_H}{L^u_H}-1\right)-\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{1-\pi^u_{HH}}{\pi^u_{HH}}\right)\right]}. \tag{D.33}$$

Recap of Key Equations

$$
\begin{aligned}
1+\tau_H^d &= \frac{\sigma}{\sigma-1}+\frac{\mu_{SB}}{\mu_d}\frac{1}{\sigma-1}\frac{1}{Q_{FH}^d}\frac{P_{HF}^u}{P_{HF}^d}\\
\frac{1+t_H^d}{1+t_H^u} &= 1+\gamma^d+\frac{\mu_{SB}}{\mu_d}\frac{\pi_{HH}^u}{Q_{HH}^u}\left[\gamma^d(1-\alpha)+\left(\frac{1}{\theta}-\alpha\right)\right]+\frac{\mu_{LC}}{\mu_c}\frac{\alpha}{1-\alpha}\frac{\theta-1}{\theta}\frac{1}{L_H^d}\frac{P_{HF}^d}{P_{HF}^u}\\
\frac{1+\tau_H^d}{1+t_H^u}-\frac{\theta-1}{\theta}\left(1+\tau_H^d\right) &= \frac{\mu_{SB}}{\mu_d}\frac{1}{\theta}\left[\frac{1}{Q_{HF}^u}+\frac{1}{Q_{HH}^u}\right]-\frac{\mu_{LC}}{\mu_d}\frac{\alpha}{1-\alpha}\frac{1}{L_H^d}\frac{1}{\theta}\frac{1}{\pi_{HH}^u}\frac{P_{HF}^d}{P_{HF}^u}\\
\frac{\mu_{LC}}{\mu_d} &= \frac{\frac{(1+\gamma^d)\gamma^u}{(1+\gamma^u)}X_H^d+\frac{\mu_{SB}}{\mu_d}(1-\alpha)\frac{X_H^d\pi_{HH}^u}{Q_{HH}^u}\left[\frac{(1+\gamma^d)}{(1+\gamma^u)}\gamma^u-\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{(1-\pi_{HH}^u)}{\pi_{HH}^u}\right)\right]}{\bar{A}_H^u\left(L_H^u\right)^{\gamma^u}\left[\frac{1}{\alpha}\frac{1}{1+\gamma^u}\left(\frac{\gamma^u L_H^d}{L_H^u}-1\right)-\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{1-\pi_{HH}^u}{\pi_{HH}^u}\right)\right]}
\end{aligned}
$$

We have not been successful in proving any general results, so let us study some special cases.

B. Second-Best Import Tariffs with No Scale Economies

Given the above derivations, it is straightforward to prove that Proposition 7 applies even when $\alpha > 0$. Simply set $\gamma^d = \gamma^u = 0$ in equations (D.31), (D.32)and (D.33). First note, equation (D.33) becomes

$$
\frac{\mu_{LC}}{\mu_d}=\frac{\mu_{SB}}{\mu_d}\frac{(1-\alpha)\frac{X_H^d\pi_{HH}^u}{Q_{HH}^u}\left[\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{(1-\pi_{HH}^u)}{\pi_{HH}^u}\right)\right]}{\bar{A}_H^u\left(L_H^u\right)^{\gamma^u}\left[\frac{1}{\alpha}+\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{1-\pi_{HH}^u}{\pi_{HH}^u}\right)\right]},
$$

and plugging (D.30),

$$
\frac{\mu_{LC}}{\mu_d}=\frac{\mu_{SB}}{\mu_d}(1-\alpha)^2 X_H^d L_H^d\left(\frac{\pi_{HH}^u}{Q_{HH}^u}\right)^2\frac{1+\frac{1}{\theta}\frac{(1-\pi_{HH}^u)}{\pi_{HH}^u}}{1+\frac{\alpha}{\theta}\frac{1-\pi_{HH}^u}{\pi_{HH}^u}}.
$$

Plugging this expression for $\frac{\mu_{LC}}{\mu_d}$ into (D.31), delivers

$$
\frac{1+t_H^d}{1+t_H^u}=1+\frac{\mu_{SB}}{\mu_d}\frac{\pi_{HH}^u}{Q_{HH}^u}(1-\alpha)\left[\frac{\alpha+\pi_{HH}^u(1-\alpha)}{\alpha+\pi_{HH}^u(\theta-\alpha)}\right]
$$

And finally, plugging $\frac{\mu_{LC}}{\mu_d}$ into equation (D.32) delivers

$$
\frac{1+\tau_H^d}{1+t_H^u}-\frac{\theta-1}{\theta}\left(1+\tau_H^d\right)=\frac{\mu_{SB}}{\mu_d}\frac{1}{\theta}\left[\frac{1}{Q_{HF}^u}+\frac{1}{Q_{HH}^u}\frac{(1-\alpha)\theta\pi_{HH}^u}{\alpha+\pi_{HH}^u(\theta-\alpha)}\right].
$$

In sum, we can write the system as

$$
\begin{aligned}
1+\tau_{FH}^d &= \frac{\sigma}{\sigma-1}+\frac{\mu_{SB}}{\mu_d}A\\
\frac{1+\tau_{FH}^d}{1+t_H^u} &= 1+\frac{\mu_{SB}}{\mu_d}B\\
\frac{1+\tau_H^d}{1+t_H^u}-\frac{\theta-1}{\theta}\left(1+\tau_H^d\right) &= \frac{\mu_{SB}}{\mu_d}C
\end{aligned}
$$

where

$$
\begin{aligned}
A &= \frac{1}{\sigma-1}\frac{1}{C_{HF}}\frac{P_{HF}^u}{P_{HF}^d} > 0\\
B &= \frac{\pi_{HH}^u}{Q_{HH}^u}(1-\alpha)\left[\frac{\alpha+\pi_{HH}^u(1-\alpha)}{\alpha+\pi_{HH}^u(\theta-\alpha)}\right] > 0\\
C &= \frac{1}{\theta}\left[\frac{1}{Q_{HF}^u}+\frac{1}{Q_{HH}^u}\frac{(1-\alpha)\theta\pi_{HH}^u}{\alpha+\pi_{HH}^u(\theta-\alpha)}\right] > 0
\end{aligned}
$$

So we have

$$
\begin{aligned}
1+\tau_{FH}^d &= \frac{\sigma}{\sigma-1}+\left[\frac{1+\tau_{FH}^d}{1+t_H^u}-\frac{\theta-1}{\theta}\left(1+\tau_{FH}^d\right)\right]\frac{A}{C}\\
\frac{1+\tau_{FH}^d}{1+t_H^u} &= 1+\left(\frac{1+\tau_{FH}^d}{1+t_H^u}-\left(1+\tau_{FH}^d\right)\frac{\theta-1}{\theta}\right)\frac{B}{C}
\end{aligned}
$$

When solving this system, we obtain

$$
\frac{1+\tau_{FH}^d}{1+t_H^u} = \frac{1-\frac{\sigma}{\sigma-1}\frac{\theta-1}{\theta}\frac{B}{C}+\frac{\theta-1}{\theta}\frac{A}{C}}{1-\frac{B}{C}+\frac{\theta-1}{\theta}\frac{A}{C}},
$$

which is higher or lower than 1 depending on the relative size of σ and θ. More specifically, when $\sigma > \theta$, $\frac{\sigma}{\sigma-1}\frac{\theta-1}{\theta} < 1$, and we have tariff escalation. But when $\sigma < \theta$, then $\frac{\sigma}{\sigma-1}\frac{\theta-1}{\theta} > 1$, and we have tariff de-escalation.

C. Second-Best Import Tariffs with No Scale Economies Upstream ($\gamma^u = 0$)

We next study the case in which $\gamma^d > 0$ but $\gamma^u = 0$. In that case, equation (D.33) reduces to

$$
\frac{\mu_{LC}}{\mu_d} = \frac{\mu_{SB}}{\mu_d}\frac{(1-\alpha)\frac{X_H^d\pi_{HH}^u}{Q_{HH}^u}\left[\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{(1-\pi_{HH}^u)}{\pi_{HH}^u}\right)\right]}{\bar{A}_H^u\left[\frac{1}{\alpha}+\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{1-\pi_{HH}^u}{\pi_{HH}^u}\right)\right]},
$$

and we can write

$$
\begin{aligned}
1+\tau_H^d &= \frac{\sigma}{\sigma-1}+\frac{\mu_{SB}}{\mu_d}A\\
\frac{1+t_H^d}{1+t_H^u} &= 1+\gamma^d+\frac{\mu_{SB}}{\mu_d}B\\
\frac{1+\tau_H^d}{1+t_H^u}-\frac{\theta-1}{\theta}\left(1+\tau_H^d\right) &= \frac{\mu_{SB}}{\mu_d}C
\end{aligned}
$$

with

$$
\begin{aligned}
B &= \frac{\pi^u_{HH}}{Q^u_{HH}}\left[\left[\gamma^d(1-\alpha)+\left(\frac{1}{\theta}-\alpha\right)\right]+\frac{X^d_H\left[\left(1+\frac{1}{\theta}\frac{(1-\pi^u_{HH})}{\pi^u_{HH}}\right)\right]}{\bar{A}^u_H\left[\frac{1-\alpha}{\alpha}+\left(1+\frac{1}{\theta}\frac{1-\pi^u_{HH}}{\pi^u_{HH}}\right)\right]}\alpha\frac{\theta-1}{\theta}\frac{1}{L^d_H}\frac{P^d_{HF}}{P^u_{HF}}\right] \\
&= \frac{\pi^u_{HH}}{Q^u_{HH}}\left[\left[\gamma^d(1-\alpha)+\left(\frac{1}{\theta}-\alpha\right)\right]+\frac{\left[\left(1+\frac{1}{\theta}\frac{(1-\pi^u_{HH})}{\pi^u_{HH}}\right)\right]}{\left[\frac{1-\alpha}{\alpha}+\left(1+\frac{1}{\theta}\frac{1-\pi^u_{HH}}{\pi^u_{HH}}\right)\right]}\frac{\theta-1}{\theta}\right] \\
&= \frac{\pi^u_{HH}}{Q^u_{HH}}(1-\alpha)\frac{\pi^u_{HH}(\theta-1)+\alpha\sigma+\pi^u_{HH}\sigma(1-\alpha)}{(\sigma-1)\left(\alpha(1-\pi^u_{HH})+\pi^u_{HH}\theta\right)} > 0.
\end{aligned}
$$

and

$$
C = \frac{1}{\theta}\frac{1}{Q^u_{HF}}+\frac{1}{Q^u_{HH}}\frac{1}{\theta}\left[1-\frac{\left(1+\frac{1}{\theta}\frac{(1-\pi^u_{HH})}{\pi^u_{HH}}\right)}{\left[\frac{1-\alpha}{\alpha}+\left(1+\frac{1}{\theta}\frac{1-\pi^u_{HH}}{\pi^u_{HH}}\right)\right]}\right] > 0.
$$

Given $B > 0$ and $C > 0$, it is then straightforward to use the same steps as in the proof of the $\alpha = 0$ case in Online Appendix C.4 to show that

$$
1+\tau^d_H > \frac{\sigma}{\sigma-1}
$$

and

$$
\frac{1+t^d_H}{1+t^u_H} > 1+\gamma^d.
$$

E Data Appendix

E.1 Data Construction for Figure 2

US Tariff Data.

- We use US import tariff data at the 8-digit level from the US Harmonized Tariff Schedule (HTS) available at https://dataweb.usitc.gov/tariff/annual. We use the most-favored-nation (MFN) ad valorem tariff rate whenever possible. In approximately 25% of the cases, the MFN ad valorem rate is not available and instead a "specific" tariff rate is applied such as "68 cents/head", "1 cents/kg", "0.9 cents each" etc. In these cases we perform an imputation by calculating an ad valorem equivalent tariff rate using unit values obtained from the US Census Bureau.

- In a next step we use the imputed ad valorem tariff rate to calculate applied MFN ad valorem tariff rates for all goods, taking trade agreements between the US and other countries into account. This is, we calculate the applied MFN ad valorem tariff rate as an import weighted average of the MFN ad-valorem rate and the tariff rate that is paid by countries that are members of a trade agreement.[5] US import data for the year 2015 come from the US Census Bureau.

- Data on tariffs imposed in February and March 2018 on almost all countries (washers; solar panels; iron and steel; aluminum) come from Fajgelbaum et al. (2020) and all subsequent tariffs imposed on imports from China throughout 2018 and 2019 from Chad Bown (available here).

ROW Tariff Data.

- We use tariff data for 115 countries plus the European Union at the 6-digit HS code level from the WTO Tariff Download Facility available at http://tariffdata.wto.org/default.aspx. We use the most-favored-nation (MFN) ad valorem tariff rate which constitutes the simple average duty of all products within a 6-digit HS code classification.

- We use data on retaliatory tariffs imposed by China throughout 2018 and 2019 from Chad Bown (available here). Data on retaliatory tariffs imposed by the European Union, Canada, Mexico, India and Turkey stem from Li (2018). Using data on these tariff waves we adjust the MFN applied tariff rates taking 2015 US export value weighted averages with US export data coming from the US Census Bureau.

Intermediate and Final Goods Classification

- We classify goods into intermediate and final goods using the UN Broad Economic Categories (BEC). The cross-walk between HTS10 codes and end-use categories is available here). We classify goods as intermediate goods when their BEC code starts with 111, 121, 21, 22, 31, 322, 42 and 53. Final goods (including capital goods) start with BEC code 41, 521, 112, 321, 522, 61, 62, 63. All other goods have no classification.

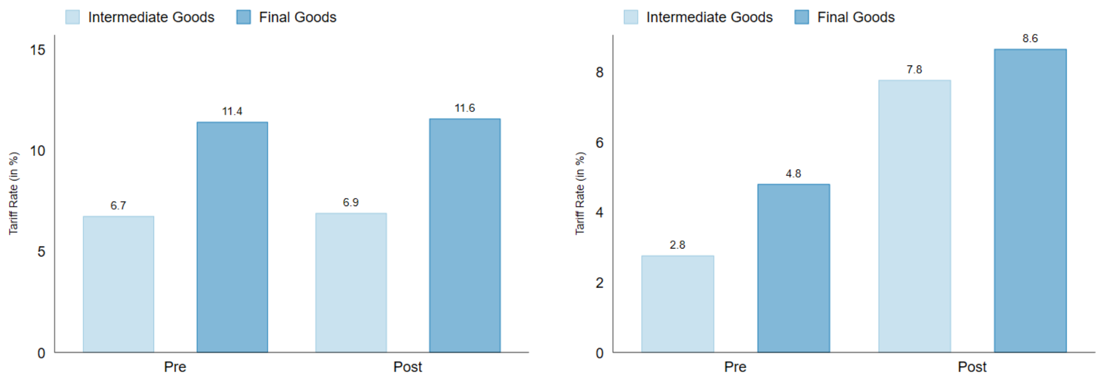
[5] We currently account for the following trade agreements: Generalized System of Preferences (GSP, 41 countries), The Agreement on Trade in Civil Aircraft (32 countries), NAFTA (3 countries), Caribbean Basin Initiative (CBI, 17 countries), African Growth and Opportunity Act (AGOA, 40 countries), Caribbean Basin Trade Partnership Act (CBTPA, 8 countries), Dominican Republic-Central America FTA (6 countries) and the Agreement on Trade in Pharmaceutical Products (7 countries).

Tariff Escalation Unweighted

- As alternative to Figure 2 which shows trade-weighted tariff rates, Figure E.1 displays an unweighted version of the tariff increase on intermediate and final goods by the ROW on imports from the US throughout the trade war and vice versa.

Figure E.1: Comparison ROW and US (Unweighted)

(a) ROW average tariffs on US **(b)** US average tariffs on ROW



Notes: Pre: Tariffs in January 2018, Post: Tariffs in December 2019. Tariff data from WTO and USITC. Goods are classified as intermediate goods when their BEC code starts with 111, 121, 21, 22, 31, 322, 42 and 53. Final goods start with BEC code 41, 521, 112, 321, 522, 61, 62, 63 (including capital goods). All other goods have no classification.

E.2 Elasticity Estimation

We now explain the estimation of elasticities of substitution in the upstream and downstream sectors using three different approaches: trade elasticity approach, sectoral markup approach and scale elasticity approach. We present results for all three approaches and demonstrate how they differ.

Sectoral Markup Approach Our first elasticity estimation approach relies on sectoral markups. Information on firm-level markups allow us to derive elasticities in a straightforward manner since equations (A.1) and (A.2) illustrate that $markup = \frac{elasticity}{elasticity-1}$. We thus compile data for this exercise as follows:

We obtain upstream/downstream sector classifications using WIOD. We use 2014 sales of the US to the US and RoW to calculate the share of total sales per sector that goes to final consumers. We then classify a sector as upstream when the share of total sales to final consumers is below the median across all sectors and as downstream when the share is above the median. This yields a dataset which shows upstream and downstream classifications for 87 sectors at the 2-digit NACE level (European industry classification). This 2-digit NACE data we combine with a NACE-NAICS concordance file that maps 4-digit NACE (we only use the first 2 digits) to 6-digit NAICS. If there exists multiple NACE 2-digit codes for a NAICS 6-digit code we choose the NACE 2-digit code that has larger total US sales. This yields a final dataset that shows upstream and downstream classifications for 1,175 different NAICS 6-digit codes. This dataset we combine with data kindly provided by Baqaee and Farhi (2020) (BF) based on 6-digit NAICS codes. The BF data lists markups and sales for 31,683 different firms from 1978 âĂŞ 2018. They provide three different types of markups calculated based on a user cost / production function / or accounting profits method. We select their data between 2012 and 2017 and focus on the markups calculated using the production function estimation approach. We further exclude firms that have markups smaller than 1 (14% of all firm-year observations).

Table E.1: Elasticities

	mean	sd	min	p5	p25	p50	p75	p95	max	count
Upstream	4.43	4.26	1.10	1.15	1.60	2.75	5.04	16.50	16.50	11045
Downstream	6.44	6.05	1.29	1.46	2.44	4.03	7.49	22.24	22.24	14773

Notes: The table shows weighted mean elasticities for upstream and downstream sectors between 2012 and 2017 across all firms in the WIOD that have markups greater than 1. Elasticities stem from the production function estimation approach. Weights represent the share of firm sales in total sales. We winsorize elasticities and sales at the 5-95th percentile by sector.

We then calculate firm-level elasticities as $elasticity = \frac{markup}{markup-1}$ and winsorize elasticities and sales at the 5-95th percentile by sector. Finally, we calculate weighted mean elasticities for upstream and downstream sectors across all firms where weights represent the share of firm sales in total sales. Table E.1 presents elasticities for upstream and downstream sectors pooling all years from 2012 to 2017.

Trade Elasticity Approach In our second elasticisty estimation approach, we estimate elasticities in the upstream and downstream sectors by measuring the response of imports in the up-and downstream sectors to changes in import tariffs. More specifically, we calculate the changes in US import values in both sectors during the US-China trade war (January 2018 to December 2019) that raised US import tariffs on upstream goods by 4.1 percentage points and downstream goods by 4.4 percentage points. We obtain data on import values at the country-HTS10-month level from the US Census Bureau's Application Programming Interface (API). Data on US import tariffs is constructed as described in Section E.1.

We regress 12-month log changes in import values on 12-month log changes in tariff rates via the following regression specification:

$$\Delta ln(v_{ijt}) = \alpha_j + \tau_{it} + \beta \Delta ln(1 + Tariff_{ijt}) + \omega_{ijt} \tag{E.1}$$

where i indicates foreign countries, j denotes products and t corresponds to time; α_j is a product fixed effect; τ_{it} is a country-time fixed effect; and ω_{ijt} is a stochastic error. We denote import values by v_{ijt}. We estimate equation E.1 separately for intermediate and final goods using both log differences and the inverse of the hyperbolic sine transformation, $log[x + (x^2 + 1)^{0.5}]$, to be able to estimate changes when import values are zero in t or $t - 12$.[6] The results are presented in Table E.2.

Column 1 (3) suggests that a one percent increase in tariffs on intermediate (final) goods is associated with a 1.05 (1.81) percent decrease in import value. However, since tariffs can lead to zero imports, which will be dropped from the regression, columns 2 and 4 perform the same regression this time using the inverse hyperbolic sine instead of the log change. This adjustments leads to greater trade elasticities for both types of goods. A one percent increase in tariffs on intermediate (final) goods is associated with a 2.35 (3.08) percent decrease in import value. Note that the estimates from this specification correspond to an elasticity of substitution between intermediate (final) goods of 2.35 (2.08).

[6]Note that regression coefficients based on the hyperbolic sine transformation are sensitive to the scale of the import values. This is, results vary depending on whether import values are measured in thousands, millions, etc. Following Amiti et al. (2019), we measure import values in single US dollars.

Table E.2: Impact of US Tariffs on Import Values

	Intermediate Goods		Final Goods	
	(1) Log Change Import Value $\Delta ln(v_{ijt})$	(2) Inv. Hyperb. Import Value $\Delta ln(v_{ijt})$	(3) Log Change Import Value $\Delta ln(v_{ijt})$	(4) Inv. Hyperb. Import Value $\Delta ln(v_{ijt})$
log change tariff				
$\Delta ln(1 + Tariff_{ijt})$	-1.05***	-2.35***	-1.81***	-3.08***
	(0.07)	(0.44)	(0.08)	(0.35)
N	1302744	2220920	1253577	2251844
R2	0.027	0.048	0.022	0.045

Notes: Observations are at the country-HTS10-month level for the period January 2018 to December 2019. Since the specification is in 12-month changes, the data includes obesrvations from January 2017 onwards. Robust standard errors in parentheses. Variables are in twelve-month log change. All columns include product-level and country-time fixed effects. The dependent variables are the log change and the change in the inverse hyperbolic sine of US import values of intermediate and final goods, respectively. We use the inverse of the hyperbolic sine transformation, $log[x + (x^2 + 1)^{0.5}]$, to be able to estimate changes when import values are zero in t or $t - 12$. *$p < 0.05$, **$p < 0.01$, ***$p < 0.001$.

Scale Elasticity Approach Our final elasticity estimation approach exploits the isomorphism of our model to a model with external economies of scale. As discussed in Section 2, these models are isomorphic provided that the following restrictions between the external economies of scale parameters and the elasticities of substitution across varieties hold: $\gamma^u = 1/(\theta - 1)$ and $\gamma^d = 1/(\sigma - 1)$. Data on γ^u and γ^d thus allow us to easily derive information on elasticities.

Data on scale elasticities comes from Bartelme et al. (2019). The authors provide 2SLS estimates on scale elasticities for 15 manufacturing industries presented in Table E.3. We classify these industries into upstream and downstream industries following the same procedure as in the *Sectoral Markup Approach* and then calculate the average scale elasticity in those sectors.

Table E.3: Scale Elasticities

Industry	NACE Rev. 2	WIOD class.	Scale elast.
Food products, beverages and tobacco	10, 11, 12	downstream	0.16
Textiles	13, 14, 15	downstream	0.12
Wood and products of wood and cork	16	upstream	0.11
Paper products and printing	17, 18	upstream	0.11
Coke and refined petroleum products	19	upstream	0.07
Chemicals and pharmaceutical products	20, 21	upstream	0.2
Rubber and plastic products	22	upstream	0.25
Other non-metallic mineral products	23	upstream	0.13
Basic metals	24	upstream	0.11
Fabricated metal products	25	upstream	0.13
Computer, electronic and optical products	26	downstream	0.13
Electrical equipment	27	upstream	0.09
Machinery and equipment, nec	28	downstream	0.09
Motor vehicles, trailers and semi-trailers	29	downstream	0.15
Other transport equipment	30	downstream	0.16

Notes: Industries and 2SLS scale elasticities stem from Bartelme et al. (2019). Upstream and downstream classifications stem from WIOD where we classify a sector as upstream when the share of total sales to final consumers is below the median across all sectors and as downstream when the share is above the median.

For the upstream sector we obtain an average scale elasticity of 0.133 and for the downstream sector an average scale elasticity of 0.135. Exploiting the isomorphism between this setup and our framework with monopolistic competition and free entry, we convert these to $\theta = 8.52$ and $\sigma = 8.41$.

E.3 Share of Inputs in the Downstream Sector

As in the “Sectoral Markup Approach," we classify sectors into upstream and downstream depending on whether he share of total sales to final consumers is below or above the median across all sectors. From the WIOD in 2014 we calculate the share of inputs in the downstream sector as the ratio of intermediate inputs to sales in the downstream sectors leading to an estimate of $1 - \alpha = 0.45$.

F Additional Quantitative Results

F.1 Robustness

Table F.1: Calibrated Parameters - Robustness

	τ^d	τ^u	A^d_{RoW}	A^u_{RoW}
$\theta = 4.43$ and $\sigma = 6.44$	1.787	2.2986	0.289	0.114
$\theta = 2.35$ and $\sigma = 3.08$	5.021	8.536	0.248	0.102
$\theta = 8.52$ and $\sigma = 8.41$	1.508	1.446	0.301	0.121
$\theta = 2.5$ and $\sigma = 4$	3.007	6.878	0.260	0.103
$\theta = 5.5$ and $\sigma = 4$	3.007	1.877	0.281	0.116
$\alpha = 0.75$	2.249	2.040	0.196	0.114
$\alpha = 0.25$	2.239	2.042	0.119	0.124
$\alpha = 0$	2.375	2.073	0.598	0.128

Notes: This table reports the re-calibrated parameters in our robustness exercise.