

# A Theoretical Appendix

The proof of a few theoretical results in the main text of this book were only sketched or simply omitted to enhance the flow of text. In this Appendix, I provide the details of these proofs or refer the reader to the relevant papers where the proofs can be found.

## A.1 Optimal Sourcing Strategy in the Multi-Country Global Sourcing Model in Chapter 2

In this Appendix, I formally prove some statements related to the characterization of the optimal sourcing strategy in the multi-country global sourcing model in Chapter 2.

Remember that the problem of choosing an optimal sourcing strategy is given by

$$\begin{aligned} \max_{\mathcal{J}_i(\varphi)} \pi_i(\varphi, \mathcal{J}_i(\varphi)) &= (a_{hi}w_i)^{-\eta(\sigma-1)} \left( \gamma \sum_{k \in \mathcal{J}_i(\varphi)} T_k (\tau_{ik}w_k)^{-\theta} \right)^{(\sigma-1)(1-\eta)/\theta} B\varphi^{\sigma-1} \\ &\quad - w_i \sum_{k \in \mathcal{J}_i(\varphi)} f_{ik}. \end{aligned}$$

With a discrete number of locations, we can rewrite the problem as follows:

$$\begin{aligned} \max_{I_1, I_2, \dots, I_J \in \{0,1\}^J} \pi_i(\varphi, I_1, I_2, \dots, I_J) &= \left( \sum_{k=1}^J I_k T_k (\tau_{ik}w_k)^{-\theta} \right)^{(\sigma-1)(1-\eta)/\theta} \tilde{B}\varphi^{\sigma-1} \\ &\quad - w_i \sum_{k=1}^J I_k f_{ik}, \end{aligned} \tag{A.1}$$

where  $\tilde{B} = (a_{hi}w_i)^{-\eta(\sigma-1)} \gamma^{(\sigma-1)(1-\eta)} B$ . The dummy variable  $I_j$  thus takes a value of 1 when  $j \in \mathcal{J}_i(\varphi)$  and 0 otherwise.

The key thing to note is that, provided that  $(\sigma-1)(1-\eta) > \theta$ , the modified objective function in (A.1) features increasing differences in  $(I_j, I_k)$  for  $j, k \in \{1, \dots, J\}$  such that  $j \neq k$ , and also features increasing differences in  $(I_j, \varphi)$  for

any  $j \in \{1, \dots, J\}$ . Invoking standard results in monotone comparative statics, we can then conclude that for  $\varphi_1 \geq \varphi_0$ , we must have  $(I_1^*(\varphi_1), I_2^*(\varphi_1), \dots, I_J^*(\varphi_1)) \geq (I_1^*(\varphi_0), I_2^*(\varphi_0), \dots, I_J^*(\varphi_0))$ . Naturally, this rules out a situation in which  $I_j^*(\varphi_1) = 0$  but  $I_j^*(\varphi_0) = 1$ , and thus we can conclude that  $\mathcal{J}_i(\varphi_0) \subseteq \mathcal{J}_i(\varphi_1)$  for  $\varphi_1 \geq \varphi_0$ .

## A.2 Comparative Statics of the Global Sourcing Model in Chapter 4

In this Appendix, I will provide formal proofs for some comparative statics mentioned in Chapters 4 and 5. When a result has been proven in an existing paper, I will simply refer the reader to that paper.

### Derivation of a General Formula for the Offshoring Share

In the first part of Chapter 5, I studied the determinants of the cross-section of offshoring shares. In Chapter 4, I derived a formula for this share but under the strong assumptions of complete contracting in the North, ‘totally’ incomplete contracting in the South, a single input, and symmetric bargaining. In Chapter 5, I appealed to a general formula that applied to all the extensions of the two-country model developed in Chapter 4. Let me now provide more details on that derivation.

As explained in Chapter 4, the share of foreign input purchases in total input purchases typically depends on how these inputs are priced in the presence of incomplete contracting and renegotiation. Below, I stick to the assumption in the main text that the ratio of input expenditures to sale revenue is common for firms sourcing domestically and offshoring. As a result, the offshoring share is identical to the fraction of industry sales captured by firms offshoring intermediate inputs. With a constant price elasticity of demand  $\sigma > 1$ , firm revenues are in turn a multiple  $\sigma$  of operating profits. Operating profits are in turn equal to overall profits plus fixed costs, or

$$\begin{aligned}\pi_D(\varphi) + f_D w_N &= (w_N)^{1-\sigma} B \Gamma_D \varphi^{\sigma-1} \\ \pi_O(\varphi) + f_O w_N &= \left( (w_N)^\eta (\tau w_S)^{1-\eta} \right)^{1-\sigma} B \Gamma_O \varphi^{\sigma-1}.\end{aligned}$$

Assuming selection into offshoring – i.e., condition (2.21) in Chapter 2 –, we can define the thresholds  $\tilde{\varphi}_O > \tilde{\varphi}_D$  satisfying  $\pi_D(\tilde{\varphi}_D) = 0$  and  $\pi_O(\tilde{\varphi}_O) = \pi_D(\tilde{\varphi}_O)$ . It is straightforward to verify that

$$\frac{\tilde{\varphi}_O}{\tilde{\varphi}_D} = \left[ \frac{f_O/f_D - 1}{\frac{\Gamma_O}{\Gamma_D} \left( \frac{w_N}{\tau w_S} \right)^{(1-\eta)(\sigma-1)} - 1} \right]^{1/(\sigma-1)}. \quad (\text{A.2})$$

The share of revenues (and of input purchases) accounted for by offshoring firms is then given by

$$\Upsilon_O = \frac{\int_{\tilde{\varphi}_O}^{\infty} \left( (w_N)^\eta (\tau w_S)^{1-\eta} \right)^{1-\sigma} B \Gamma_O \varphi^{\sigma-1} dG(\varphi)}{\int_{\tilde{\varphi}_D}^{\tilde{\varphi}_O} (w_N)^{1-\sigma} B \Gamma_D \varphi^{\sigma-1} dG(\varphi) + \int_{\tilde{\varphi}_O}^{\infty} \left( (w_N)^\eta (\tau w_S)^{1-\eta} \right)^{1-\sigma} B \Gamma_O \varphi^{\sigma-1} dG(\varphi)}.$$

Assuming a Pareto distribution of productivity – i.e.,  $G(\varphi) = 1 - (\underline{\varphi}/\varphi)^\kappa$  for  $\varphi \geq \underline{\varphi} > 0$  –, this expression further simplifies to

$$\Upsilon_O = \frac{\frac{\Gamma_O}{\Gamma_D} \left( \frac{w_N}{\tau w_S} \right)^{(1-\eta)(\sigma-1)}}{\left( \frac{\tilde{\varphi}_O}{\tilde{\varphi}_D} \right)^{\kappa-(\sigma-1)} - 1 + \frac{\Gamma_O}{\Gamma_D} \left( \frac{w_N}{\tau w_S} \right)^{(1-\eta)(\sigma-1)},}$$

where  $\tilde{\varphi}_O/\tilde{\varphi}_D$  is given in A.2. This corresponds to the general offshoring share equation (5.2) in Chapter 5. This formula is identical to the one applying in the complete-contracting case except for the term  $\Gamma_O/\Gamma_D$ .

With this expression in hand, we can next turn to the study of comparative statics in the different variants of the global sourcing model developed in Chapter 4. Below I will focus on how the different parameters of the model shape the ratio  $\Gamma_O/\Gamma_D$ , which differs across variants of the model. As argued in the main text, leaving aside this term  $\Gamma_O/\Gamma_D$ , the share  $\Upsilon_O$  is increasing in  $w_N/w_S$  and  $\sigma$ , and decreasing in  $\tau$ ,  $f_O/f_D$ ,  $\kappa$  and  $\eta$  (these results are straightforward to prove by simple differentiation (making use of  $\kappa \geq \sigma - 1$ )).

## Symmetric Nash Bargaining Model

Consider first the basic model with complete contracting in the North, ‘totally’ incomplete contracting in the South, a single input, and symmetric bargaining. This implies  $\Gamma_D = 1$  and thus (see eq. (4.10)).

$$\frac{\Gamma_O}{\Gamma_D} = (\sigma + 1) \left( \frac{1}{2} \right)^\sigma.$$

But note that

$$\frac{\partial (\Gamma_O/\Gamma_D)}{\partial \sigma} = - \left( \frac{1}{2} \right)^\sigma ((1 + \sigma) \ln 2 - 1) < 0,$$

and thus the offshoring share is lower in higher elasticity sectors on account of the effect of contractual frictions. This effect is of the opposite sign to the ‘standard’ one operating in the complete-contracting case, and thus the overall effect of  $\sigma$  on the offshoring share  $\Upsilon_O$  is ambiguous.

## Generalized Nash Bargaining Model

Let us now turn to the basic model with generalized Nash bargaining. Again we have  $\Gamma_D = 1$  and thus

$$\frac{\Gamma_O}{\Gamma_D} = \Gamma_\beta \equiv (\sigma - (\sigma - 1)(\beta\eta + (1 - \beta)(1 - \eta))) \left( \beta^\eta (1 - \beta)^{1-\eta} \right)^{\sigma-1},$$

as indicated by equation (4.14).

As mentioned in the main text, the effects of  $\beta$  and  $\eta$  on  $\Gamma_\beta$  are ambiguous and interact with each other. More specifically, we next show that  $\Gamma_\beta$  is decreasing in  $\eta$  when  $\beta < 1/2$ , while it is increasing in  $\eta$  when  $\beta > 1/2$ . To see this, first note that

$$\left. \frac{\partial \ln \Gamma_\beta}{\partial \eta} \right|_{\eta=0} = (\sigma - 1) \left( \frac{1 - 2\beta}{1 - \beta + \sigma\beta} + \ln \left( \frac{\beta}{1 - \beta} \right) \right)$$

and

$$\left. \frac{\partial \ln \Gamma_\beta}{\partial \eta} \right|_{\eta=1} = (\sigma - 1) \left( \frac{1 - 2\beta}{\sigma(1 - \beta) + \beta} + \ln \left( \frac{\beta}{1 - \beta} \right) \right).$$

It is not hard to show that each of these two expressions is negative for  $\beta < 1/2$  and positive for  $\beta > 1/2$ . In particular, one can use  $1 - x + \ln x \leq 0$  and  $\ln 1/x - (1 - x) \geq 0$ , with  $x = \beta/(1 - \beta)$  to rewrite these expressions in a way that makes this obvious by inspection.<sup>24</sup> Next, notice that

$$\frac{\partial^2 \ln \Gamma_\beta}{\partial \eta^2} = - \frac{(\sigma - 1)^2 (1 - 2\beta)^2}{(\sigma - (\sigma - 1)(\beta\eta + (1 - \beta)(1 - \eta)))^2} < 0.$$

In sum, we have that when  $\beta < 1/2$ ,  $\partial \ln \Gamma_\beta / \partial \eta < 0$  when evaluated at  $\eta = 0$ , while for  $\beta > 1/2$ ,  $\partial \Gamma_\beta / \partial \eta > 0$  when evaluated at  $\eta = 1$ . Together with the concavity of  $\Gamma_\beta$ , we can then conclude that  $(\partial \ln \Gamma_\beta / \partial \eta)(\beta - 1/2) \geq 0$  for all  $\eta$ , with strict inequality for  $\beta \neq 1/2$ . The practical relevance of this result is that it complicates the overall comparative static of the offshoring share  $\Upsilon_O$  with respect to  $\eta$  (remember that under complete contracting,  $\Upsilon_O$  is unambiguously decreasing in  $\eta$ ).

We next consider how the ambiguous effect of changes in  $\beta$  interacts with  $\eta$ . I begin by noting that simple differentiation delivers

$$\frac{\partial \ln (\Gamma_O / \Gamma_D)}{\partial \beta} = (\sigma - 1) \frac{\eta(1 - \eta) + \sigma\eta^2 - ((\sigma - 1)\eta + 1)2\eta\beta + \sigma(2\eta - 1)\beta^2}{\beta(1 - \beta)(\sigma - (\sigma - 1)(\beta\eta + (1 - \beta)(1 - \eta)))}$$

<sup>24</sup>For completeness, note that  $\frac{1-2\beta}{1-\beta+\sigma\beta} + \ln\left(\frac{\beta}{1-\beta}\right) = (2\beta-1)\frac{\sigma\beta}{(1-\beta)(1-\beta+\sigma\beta)} + \left(1 - \frac{\beta}{1-\beta}\right) + \ln\left(\frac{\beta}{1-\beta}\right) = (2\beta-1)\frac{(\sigma-2)\beta+1}{\beta(\sigma-1)\beta+1} + \ln\left(\frac{\beta}{1-\beta}\right) - \left(1 - \frac{1-\beta}{\beta}\right)$  and  $\frac{1-2\beta}{\sigma(1-\beta)+\beta} + \ln\left(\frac{\beta}{1-\beta}\right) = (2\beta-1)\frac{\sigma-1-(\sigma-2)\beta}{(1-\beta)(\sigma+\beta-\sigma\beta)} + \left(1 - \frac{\beta}{1-\beta}\right) + \ln\left(\frac{\beta}{1-\beta}\right) = \sigma(2\beta-1)\frac{1-\beta}{\beta(\sigma+\beta-\sigma\beta)} + \ln\left(\frac{\beta}{1-\beta}\right) - \left(1 - \frac{1-\beta}{\beta}\right)$ .

and

$$\frac{\partial^2 \ln(\Gamma_O/\Gamma_D)}{\partial \beta^2} = -(\sigma - 1)^2 \left( \frac{(2\eta - 1)^2}{(\sigma - (\sigma - 1)(\beta\eta + (1 - \beta)(1 - \eta)))^2} + \frac{\eta(1 - \beta) + \beta(\beta - \eta)}{(\sigma - 1)\beta^2(1 - \beta)^2} \right) < 0.$$

Thus,  $\Gamma_O/\Gamma_D$  is maximized for the value(s) of  $\beta$  that solve the quadratic equation in the numerator of  $\partial \ln(\Gamma_O/\Gamma_D)/\partial \beta$ . It turns out that there is only one solution  $\beta^*$  of this quadratic equation satisfying  $\beta^* \in [0, 1]$ . Rearranging this solution, we find equation (4.15) in Chapter 4, which makes it clear that  $\beta^*$  is increasing in  $\eta$ .

Consider finally how the elasticity of demand affects the ratio  $\Gamma_O/\Gamma_D$ . Simple (though tedious) differentiation confirms first that

$$\frac{\partial^2 (\ln(\Gamma_O/\Gamma_D))}{\partial \sigma^2} = -\frac{(\beta + \eta - 2\beta\eta)^2}{(\sigma - (\sigma - 1)(\beta\eta + (1 - \beta)(1 - \eta)))^2} < 0.$$

Hence  $\partial \ln(\Gamma_O/\Gamma_D)/\partial \sigma$  is bounded above by the value of this derivative when evaluated at the lowest possible value of  $\sigma$ , namely  $\sigma = 1$ . But note that

$$\left. \frac{\partial \ln(\Gamma_O/\Gamma_D)}{\partial \sigma} \right|_{\sigma=1} = \beta + \eta - 2\beta\eta + \ln(\beta^\eta(1 - \beta)^{1-\eta}).$$

To evaluate this expression, notice that it increases in  $\eta$  when  $\beta > 1/2$ , while it decreases in  $\eta$  when  $\beta < 1/2$ .<sup>25</sup> Furthermore, the expression equals  $\beta + \ln(1 - \beta) \leq 0$  when  $\eta = 0$ ,  $1 - \beta + \ln(\beta) \leq 0$  when  $\eta = 1$ , and  $\frac{1}{2} + \ln(\frac{1}{2}) < 0$  when  $\beta = 1/2$ . We can thus conclude that  $\partial \ln(\Gamma_O/\Gamma_D)/\partial \sigma < 0$  for  $\sigma > 1$ .

## Limitations on Ex-Ante Transfers: Financial Constraints

Remember that the case in which  $M$  cannot transfer to  $F$  ex-ante more than a share  $\phi$  of his or her ex-post rents delivered

$$\frac{\Gamma_O}{\Gamma_D} = \Gamma_\phi \equiv (\sigma + \phi - (\sigma - 1)(1 - \phi)\eta) \left(\frac{1}{2}\right)^\sigma,$$

since again we assumed  $\Gamma_D = 1$ . It is obvious from this expression that  $\Gamma_\phi$  increases in  $\phi$  and  $\eta$ , and these effects interact in a positive manner, or  $\partial^2 \Gamma_\phi / (\partial \phi \partial \eta) > 0$ . The positive effect of  $\eta$  on  $\Gamma_\phi$  again renders ambiguous the overall effect of headquarter intensity on the offshoring share  $\Upsilon_O$  (with complete contracting,  $\Upsilon_O$  is unambiguously decreasing in  $\eta$ ).

<sup>25</sup>This in turn can be shown again by applying the inequalities  $1 - x + \ln x \leq 0$  and  $\ln 1/x - (1 - x) \geq 0$  with  $x = \beta/(1 - \beta)$ , and decomposing  $1 - 2\beta + \ln \frac{\beta}{1-\beta} = \beta \frac{2\beta-1}{1-\beta} + 1 - \frac{\beta}{1-\beta} + \ln \frac{\beta}{1-\beta} = (2\beta - 1) \frac{1-\beta}{\beta} - \left(1 - \frac{1-\beta}{\beta}\right) + \ln \frac{\beta}{1-\beta}$ .

Consider next the effect of the demand elasticity  $\sigma$ . Straightforward differentiation delivers

$$\frac{\partial \ln \Gamma_\phi}{\partial \sigma} = \frac{(1 - \eta + \phi\eta)}{(\sigma + \phi - (\sigma - 1)(1 - \phi)\eta)} - \ln 2,$$

as well as  $\partial^2 \ln \Gamma_\phi / \partial \sigma^2 < 0$ . It is then straightforward to show that for a sufficiently high  $\sigma$ , we necessarily have  $\partial \ln \Gamma_\phi / \partial \sigma < 0$ . In fact, the weak condition  $\sigma + \phi > (\ln 2)^{-1} = 1.4427$  is sufficient for this inequality to hold, regardless of the value of  $\eta$ .

## Partial Contractibility

In the extension of the model with partial contractibility in both countries, I alluded to the results in Antràs and Helpman (2008) to motivate the following expressions for the index of contracting distortions under domestic sourcing and offshoring:

$$\Gamma_D = \left( \frac{\sigma}{\sigma - (\sigma - 1)(1 - \mu_N)} + 1 \right)^{\sigma - (\sigma - 1)(1 - \mu_N)} \left( \frac{1}{2} \right)^\sigma; \quad (\text{A.3})$$

$$\Gamma_O = \left( \frac{\sigma}{\sigma - (\sigma - 1)(1 - \mu_S)} + 1 \right)^{\sigma - (\sigma - 1)(1 - \mu_S)} \left( \frac{1}{2} \right)^\sigma, \quad (\text{A.4})$$

where

$$\begin{aligned} \mu_N &\equiv \eta \mu_{hN} + (1 - \eta) \mu_{mN}; \\ \mu_S &\equiv \eta \mu_{hS} + (1 - \eta) \mu_{mS}. \end{aligned}$$

In fact, these expressions are a special case of those that apply in the framework in Antràs and Helpman (2008). Because I will be referring to these more general results repeatedly in the derivations below, it might be useful to sketch here the steps that lead to that more general formula.

With that in mind, consider the following generalization of the problem in (4.19) after substitution of the participation constraint pinning down the ex-ante transfer:

$$\begin{aligned} \max_{h_c, h_n, m_c, m_n} \quad & R - w_N (\mu_{hj} h_c + (1 - \mu_{hj}) h_n) - c_j (\mu_{mj} m_c + (1 - \mu_{mj}) m_n) \\ \text{s.t.} \quad & h_n = \arg \max_h \{ \beta_h R - w_N (1 - \mu_{hj}) h_n \} \\ & m_n = \arg \max_m \{ \beta_m R - c_j (1 - \mu_{mj}) m_n \}, \end{aligned} \quad (\text{A.5})$$

where revenue is given by

$$R = B^{1/\sigma} \sigma (\sigma - 1)^{-(\sigma-1)/\sigma} \varphi^{(\sigma-1)/\sigma} \times \left( \frac{(h_c)^{\mu_{hj}} (h_n)^{1-\mu_{hj}}}{\eta} \right)^{(\sigma-1)\eta/\sigma} \left( \frac{(m_c)^{\mu_{mj}} (m_n)^{1-\mu_{mj}}}{1-\eta} \right)^{(\sigma-1)(1-\eta)/\sigma} \quad (\text{A.6})$$

and where  $c_j = w_N$  when  $j = N$  and  $c_j = \tau w_S$  when  $j = S$ . The problem above thus cover the cases of symmetric and generalized Nash bargaining, but it also encompasses environments with partial relationship-specificity in which  $F$  and  $M$  only bargain over a fraction of revenue ex-post, and thus  $\beta_h + \beta_m < 1$ . And, as discussed below, this formulation will also prove useful in the characterization of the equilibrium under multiple suppliers.

In order to derive the formula for profits associated with this more general problem, notice first that from the two constraints of the problem, we have

$$h_n = \frac{\beta_h (\sigma - 1) \eta R}{\sigma w_N}$$

$$m_n = \frac{\beta_m (\sigma - 1) (1 - \eta) R}{\sigma w_j}$$

Plugging these expressions into (A.6) delivers

$$R = \left( B^{1/\sigma} \sigma (\sigma - 1)^{-(\sigma-1)/\sigma} \varphi^{(\sigma-1)/\sigma} \right)^{\frac{\sigma}{\sigma - (\sigma-1)(1-\mu_j)}} \left( \frac{h_c}{\eta} \right)^{\frac{(\sigma-1)\eta\mu_{hj}}{\sigma - (\sigma-1)(1-\mu_j)}} \left( \frac{m_c}{1-\eta} \right)^{\frac{(\sigma-1)(1-\eta)\mu_{mj}}{\sigma - (\sigma-1)(1-\mu_j)}} \times \left( \frac{\beta_h (\sigma - 1)}{\sigma w_N} \right)^{\frac{(\sigma-1)\eta(1-\mu_{hj})}{\sigma - (\sigma-1)(1-\mu_j)}} \left( \frac{\beta_m (\sigma - 1)}{\sigma w_j} \right)^{\frac{(\sigma-1)(1-\eta)(1-\mu_{mj})}{\sigma - (\sigma-1)(1-\mu_j)}}. \quad (\text{A.7})$$

Given the Cobb-Douglas structure, we can then characterize the choice of contractible investments as satisfying

$$h_c = \frac{(\sigma - 1) \eta \left( 1 - \frac{(\sigma-1)}{\sigma} (\beta_h \eta (1 - \mu_{hj}) + \beta_m (1 - \eta) (1 - \mu_{mj})) \right)}{(\sigma - (\sigma - 1) (1 - \mu_j)) w_N} R \quad (\text{A.8})$$

$$m_c = \frac{(\sigma - 1) (1 - \eta) \left( 1 - \frac{(\sigma-1)}{\sigma} (\beta_h \eta (1 - \mu_{hj}) + \beta_m (1 - \eta) (1 - \mu_{mj})) \right)}{(\sigma - (\sigma - 1) (1 - \mu_j)) w_j} R \quad (\text{A.9})$$

As a result, operating profits are given by

$$\left( \frac{\sigma - (\sigma - 1) (\beta_h \eta (1 - \mu_{hj}) + \beta_m (1 - \eta) (1 - \mu_{mj}))}{\sigma - (\sigma - 1) (1 - \mu_j)} \right) \frac{R}{\sigma},$$

where  $R$  can be solved by plugging the above expressions (A.8) and (A.9) into (A.7). This delivers, after some manipulations

$$R = \sigma B \left( (w_N)^\eta (w_j)^{1-\eta} \right)^{1-\sigma} \varphi^{\sigma-1} (\beta_h)^{(\sigma-1)\eta(1-\mu_{hj})} (\beta_m)^{(\sigma-1)(1-\eta)(1-\mu_{mj})} \\ \times \left( \frac{\sigma - (\sigma - 1) (\beta_h \eta (1 - \mu_{hj}) + \beta_m (1 - \eta) (1 - \mu_{mj}))}{\sigma - (\sigma - 1) (1 - \mu_j)} \right)^{(\sigma-1)\mu_j},$$

and thus

$$\pi_D(\varphi) + f_D w_N = (w_N)^{1-\sigma} B \Gamma_D \varphi^{\sigma-1} \\ \pi_O(\varphi) + f_O w_N = \left( (w_N)^\eta (\tau w_S)^{1-\eta} \right)^{1-\sigma} B \Gamma_O \varphi^{\sigma-1},$$

where

$$\Gamma_\ell = \left( \frac{\sigma - (\sigma - 1) (\beta_h \eta (1 - \mu_{hj}) + \beta_m (1 - \eta) (1 - \mu_{mj}))}{\sigma - (\sigma - 1) (1 - \mu_j)} \right)^{\sigma - (\sigma - 1) (1 - \mu_j)} \\ \times (\beta_h)^{(\sigma-1)\eta(1-\mu_{hj})} (\beta_m)^{(\sigma-1)(1-\eta)(1-\mu_{mj})} \quad (\text{A.10})$$

captures the contractual frictions associated with the sourcing options  $\ell = D$  and  $\ell = O$ , which entail manufacturing in country  $j = N$  and country  $j = S$ , respectively. Setting  $\beta_h = \beta_m = 1/2$ , it is straightforward to verify that equation (A.10) reduces to equations (A.3) and (A.4) above.

Having derived these equations, we can next turn to discussing some key comparative statics. Below, I will focus on an analysis of the general formula (A.10), with the understanding that the results obtained below also apply to the particular case with  $\beta_h = \beta_m = 1/2$ . Consider first the effect of the indices of contractibility  $\mu_{hj}$  and  $\mu_{mj}$ , and their weighted average  $\mu_j$ . As shown in Antràs and Helpman (2008) (see the proof of their Proposition 1),  $\Gamma_\ell$  is necessarily non-decreasing in each of these parameters. The proof in that paper is rather cumbersome, so it may be worth offering a much simpler proof here. Consider the case of an increase in  $\mu_{hj}$  (the derivations associated with a change in  $\mu_{mj}$  are analogous). Taking logs of (A.10), differentiating and rearranging terms, we can write

$$\frac{\partial \ln \Gamma_\ell}{\partial \mu_{hj}} = \eta (\sigma - 1) (-\ln \mathcal{Q} - (1 - \mathcal{Q}) - \ln \beta_h - (1 - \beta_h)) + \mathcal{W}, \quad (\text{A.11})$$

where

$$\mathcal{Q} = \frac{\sigma - (\sigma - 1) (1 - \mu_j)}{\sigma - (\sigma - 1) (\beta_h \eta (1 - \mu_{hj}) + \beta_m (1 - \eta) (1 - \mu_{mj}))}$$

and

$$\mathcal{W} = \eta (\sigma - 1)^2 (1 - \beta_h) \frac{(1 - \eta) (1 - \mu_{mj}) (1 - \beta_m) + \eta (1 - \mu_{hj}) (1 - \beta_h)}{\sigma - (\sigma - 1) (\beta_h \eta (1 - \mu_{hj}) + \beta_m (1 - \eta) (1 - \mu_{mj}))}.$$

It is clear that the second term  $\mathcal{W}$  in (A.11) is positive, while the first one is non-negative as well because  $-\ln x - (1 - x) \leq 0$  for all  $x$ . Thus,  $\partial \ln \Gamma_\ell / \partial \mu_{hj} \geq 0$ .

It is also clear from inspection of equation (A.10) that, as stated in the main text, the effect of improvements in contractibility interacts with the headquarter intensity of production depending on the source of these changes in contractibility. Increases in  $\mu_{hj}$  will be particularly beneficial when  $\eta$  is high, while the converse is true for  $\mu_{mj}$ . For the same reason, and as in the model with totally incomplete contracting and generalized Nash bargaining, the effect of changes in headquarter intensity on  $\Gamma_\ell$  is ambiguous.

Let us now turn to the effect of the elasticity of demand  $\sigma$  on  $\Gamma_\ell$ . Tedious differentiation of  $\Gamma_\ell$  delivers

$$\frac{\partial^2 \ln \Gamma_\ell}{\partial \sigma^2} = - \frac{(1 - \mu - \eta(1 - \mu_h) \beta_h - (1 - \eta)(1 - \mu_m) \beta_m)^2}{(1 - \mu + \sigma \mu)(\sigma - (\sigma - 1)(\beta_h \eta(1 - \mu_h) + \beta_m(1 - \eta)(1 - \mu_m)))^2} < 0$$

and

$$\left. \frac{\partial \ln \Gamma}{\partial \sigma} \right|_{\sigma=1} = 1 - \mu + \eta(1 - \mu_h)(\ln \beta_h - \beta_h) + (1 - \eta)(1 - \mu_m)(\ln \beta_m - \beta_m) \leq 0.$$

To prove the negative sign in the second equation, note that this expression is maximized when  $\beta_h = \beta_m = 1$ , and at that level  $\partial \ln \Gamma / \partial \sigma|_{\sigma=1} = 1 - \mu - \eta(1 - \mu_h) - (1 - \eta)(1 - \mu_m) = 0$ . In light of these results, we can conclude that  $\partial \ln \Gamma / \partial \sigma < 0$  for all  $\sigma > 1$ , and thus contractual frictions are again aggravated by high demand elasticities in this variant of the model.

We next show how the effects of contractibility and the elasticity of demand interact with each other. In particular, differentiating  $\partial \ln \Gamma_\ell / \partial \mu_{hj}$  in (A.11) with respect to  $\sigma$ , we find:

$$\frac{\partial^2 \ln \Gamma_\ell}{\partial \mu_{hj} \partial \sigma} = \frac{1}{(\sigma - 1)} \frac{\partial \ln \Gamma_\ell}{\partial \mu_{hj}} + \eta(\sigma - 1) \frac{\partial(-\ln \mathcal{Q} - (1 - \mathcal{Q}))}{\partial \sigma} + \frac{\partial \mathcal{W}}{\partial \sigma}.$$

We have established before that the first term is non-negative. Differentiating the second and third terms, we find

$$\frac{\partial(\ln \mathcal{Q}^{-1} - 1 + \mathcal{Q})}{\partial \sigma} = \frac{(\sigma - 1) \left( \frac{(1 - \eta)(1 - \mu_m)(1 - \beta_m) + \eta(1 - \mu_h)(1 - \beta_h)}{\sigma - (\sigma - 1)(\beta_h \eta(1 - \mu_h) + \beta_m(1 - \eta)(1 - \mu_m))} \right)^2}{(\sigma - (\sigma - 1)(1 - \eta \mu_h - (1 - \eta) \mu_m))}$$

and

$$\frac{\partial \mathcal{W}}{\partial \sigma} = (1 - \beta_h) \frac{(1 - \eta)(1 - \mu_m)(1 - \beta_m) + \eta(1 - \mu_h)(1 - \beta_h)}{(\sigma - (\sigma - 1)(\beta_h \eta(1 - \mu_h) + \beta_m(1 - \eta)(1 - \mu_m)))^2},$$

and thus these terms are non-negative as well. In sum, we can conclude that  $\frac{\partial^2 \ln \Gamma_\ell}{\partial \mu_{hj} \partial \sigma} \geq 0$ , as stated in the main text. Notice that the result is *not* particular

to the special case  $\beta_h = \beta_m = 1/2$ , nor does it require  $\beta_h + \beta_m = 1$ . It is worth pointing out that it is important that we are considering the partial derivative of the logarithm of  $\Gamma_\ell$ . Computing  $\frac{\partial^2 \Gamma_\ell}{\partial \mu_{hj} \partial \sigma}$ , we find that this expression may take negative values for some parameter values. This justifies the use of logarithms of import flows in certain empirical specifications in Chapter 5, as discussed in the main text.

I have thus far focused on providing formal proofs of the results mentioned in Chapter 4, which are key for interpreting the cross-country, cross-industry results in the second part of Chapter 5. The first part of that chapter focuses on studying the determinants of the offshoring share  $\Upsilon_O$ , which in turn depend on the ratio  $\Gamma_O/\Gamma_D$ :

$$\frac{\Gamma_O}{\Gamma_D} = \frac{\left( \frac{\sigma - (\sigma - 1)(\beta_h \eta (1 - \mu_{hS}) + \beta_m (1 - \eta)(1 - \mu_{mS}))}{\sigma - (\sigma - 1)(1 - \mu_S)} \right)^{\sigma - (\sigma - 1)(1 - \mu_S)}}{\left( \frac{\sigma - (\sigma - 1)(\beta_h \eta (1 - \mu_{hN}) + \beta_m (1 - \eta)(1 - \mu_{mN}))}{\sigma - (\sigma - 1)(1 - \mu_N)} \right)^{\sigma - (\sigma - 1)(1 - \mu_N)}} \times (\beta_h)^{(\sigma - 1)\eta(\mu_{hN} - \mu_{hS})} (\beta_m)^{(\sigma - 1)(1 - \eta)(\mu_{mN} - \mu_{mS})}.$$

From the results above, it is immediate that  $\Gamma_O/\Gamma_D$  is increasing in  $\mu_S$  and its components  $\mu_{hS}$  and  $\mu_{mS}$ , and decreasing in  $\mu_N$  and its components  $\mu_{hN}$  and  $\mu_{mN}$ . Less trivially, we can also use the results above to show that  $\Gamma_O/\Gamma_D$  is decreasing in the elasticity of demand  $\sigma$  provided that contract enforcement is higher in domestic transactions vis a vis offshoring transactions. In particular, notice that

$$\frac{\partial \ln(\Gamma_O/\Gamma_D)}{\partial \sigma} = \frac{\partial \ln(\Gamma_O)}{\partial \sigma} - \frac{\partial \ln(\Gamma_D)}{\partial \sigma},$$

and provided that  $\mu_{hN} \geq \mu_{hS}$  and  $\mu_{mN} \geq \mu_{mS}$ , we can appeal to the above results  $\frac{\partial^2 \ln \Gamma_\ell}{\partial \mu_{hj} \partial \sigma} \geq 0$  and  $\frac{\partial^2 \ln \Gamma_\ell}{\partial \mu_{mj} \partial \sigma} \geq 0$  to conclude that  $\partial \ln(\Gamma_O/\Gamma_D)/\partial \sigma \leq 0$ .

Finally, it is important to emphasize that our results above do not suggest that offshoring shares will be higher for more “contractible” goods. To see this, suppose that contractibility in the South is always a fraction  $\delta < 1$  of the one in the North, so we can write  $\mu_{hS}/\mu_{hN} = \mu_{mS}/\mu_{mN} = \delta$ . For the special case,  $\beta_h = \beta_m = 1/2$ , we then have

$$\frac{\Gamma_O}{\Gamma_D} = \frac{\left( \frac{\sigma}{\sigma - (\sigma - 1)(1 - \delta \mu_N)} + 1 \right)^{\sigma - (\sigma - 1)(1 - \delta \mu_N)}}{\left( \frac{\sigma}{\sigma - (\sigma - 1)(1 - \mu_N)} + 1 \right)^{\sigma - (\sigma - 1)(1 - \mu_N)}}.$$

Increases in  $\mu_N$  can then be interpreted as overall increases in the contractibility of goods, since they affect their contractibility proportionately, regardless of the country where production takes place. It is not hard to confirm, however, that the effect of  $\mu_N$  on the expression above is non-monotonic. For instance, if one sets  $\sigma = 10$  and  $\delta = 0.9$ ,  $\Gamma_O/\Gamma_D$  is lower when  $\mu_N = 0.7$  than when either  $\mu_N = 0.5$  or  $\mu_N = 0.9$ .

## Relationship-Specificity

As discussed in the main text, this is a special case of the more general Antràs-Helpman (2008) framework with  $\beta_h = \beta_m = 1 - \epsilon/2$ , with  $\epsilon \in [0, 1]$ . The results derived above for the case of partial contractibility thus continue to apply. Improvements in contractibility are associated with larger values of  $\Gamma_\ell$ , the elasticity of demand  $\sigma$  affects  $\Gamma_\ell$  negatively, and the positive “interaction” effect  $\partial(\partial \ln \Gamma_\ell / \partial \mu_j) / \partial \sigma > 0$  continue to apply. Similarly, we have that the offshoring share is negatively impacted by the elasticity of demand  $\sigma$  on account of the term  $\Gamma_O / \Gamma_D$  (remember though that there is a positive counterbalancing effect that applies even in the complete-contracting case).

Let us then focus on the new comparative statics that emerge when introducing relationship-specificity. Consider first the direct effect of the specificity parameter  $\epsilon$ . Simple differentiation of (4.24) delivers

$$\frac{\partial \ln \Gamma_\ell(\mu_j, \epsilon)}{\partial \epsilon} = -\frac{\sigma \epsilon (\sigma - 1) (1 - \mu_j)}{(2 - \epsilon) (2(1 - \mu_j) + (2 - \epsilon) \sigma \mu_j + (\sigma - 1 + \mu_j) \epsilon)} < 0$$

and

$$\frac{\partial^2 \ln \Gamma_\ell(\mu_j, \epsilon)}{\partial \epsilon \partial \mu_j} = \frac{2\sigma^2 \epsilon (\sigma - 1)}{(2 - \epsilon) (2(1 - \mu_j) + (2 - \epsilon) \sigma \mu_j + (\sigma - 1 + \mu_j) \epsilon)^2} > 0,$$

as stated in the main text. Hence, profitability is decreasing in specificity, and improvements in contractibility are particularly profitability-enhancing at high levels of specificity. Furthermore, we can use the latter result to conclude that

$$\frac{\partial \ln(\Gamma_O / \Gamma_D)}{\partial \epsilon} = \frac{\partial \ln(\Gamma_O)}{\partial \epsilon} - \frac{\partial \ln(\Gamma_D)}{\partial \epsilon} \leq 0$$

$\mu_{hN} \geq \mu_{hS}$  and  $\mu_{mN} \geq \mu_{mS}$ . In words, whenever contract enforcement is higher in domestic transactions relative to offshore transactions, higher levels of specificity tend to be associated with lower offshoring shares  $\Upsilon_O$ .

## Multiple Inputs and Multilateral Contracting

As mentioned in the main text, the equilibrium expressions of this variant of the model are analogous to those in Antràs and Helpman (2008) whenever  $\beta_h = \beta_m = \sigma \rho / ((\sigma - 1)(1 - \eta) + \sigma \rho)$ . Plugging these values into (A.10) delivers equation (4.28). Because (4.28) is a special case of (A.10), we can conclude once again that  $\partial \Gamma_\ell(\mu_j, \rho) / \partial \mu_j \geq 0$ . Furthermore, we can also appeal to previous results to establish that  $\partial \Gamma_\ell(\mu_j, \rho) / \partial \sigma < 0$ . This latter comparative static result would appear to be complicated by the fact that  $\beta_h$  and  $\beta_m$  are now a function of  $\sigma$ . But since  $\Gamma_\ell(\mu_j, \rho)$  in (A.10) is increasing in  $\beta_h$  and  $\beta_m$ , and each of these two shares is decreasing in  $\sigma$ , we can again conclude that  $\partial \Gamma_\ell(\mu_j, \rho, \beta_h(\sigma), \beta_m(\sigma)) / \partial \sigma < 0$ .

In addition, the cross-partial derivative  $\partial (\partial \ln \Gamma_\ell (\mu_j, \rho) / \partial \mu_j) / \partial \sigma$  continues to be positive, despite the dependence of  $\beta_h$  and  $\beta_m$  on  $\sigma$ . To see this, we can just appeal to equation (A.11) and note that each of the terms in that expression is decreasing in  $\beta_h$ , which in turn decreases in  $\sigma$ . More precisely, we have that (i)  $-\ln \mathcal{Q} - (1 - \mathcal{Q})$  is decreasing in  $\mathcal{Q}$  whenever  $\mathcal{Q} < 1$ , (ii)  $\mathcal{Q}$  is indeed lower than 1 and is increasing in  $\beta_h$ , (iii)  $-\ln \beta_h - (1 - \beta_h)$  is decreasing in  $\beta_h$  for  $\beta_h < 1$ , and (iv)  $\mathcal{W}$  is decreasing in  $\beta_h$ .

We can next turn to the effects of  $\rho$  which is the new parameter introduced in this variant of the model. Simple differentiation of equation (4.28) indicates

$$\frac{\partial \ln \Gamma_\ell (\mu_j, \rho)}{\partial \rho} = \frac{(\sigma - 1)^3 (1 - \eta)^2 (1 - \mu_j)}{\rho (\rho \sigma + (\sigma - 1) (1 - \eta)) ((\sigma - (\sigma - 1) (1 - \mu_j)) \rho + (\sigma - 1) (1 - \eta))} > 0$$

and

$$\frac{\partial^2 \ln \Gamma_\ell (\mu_j, \rho)}{\partial \rho \partial \mu_j} = -(\sigma - 1)^3 \frac{(1 - \eta)^2}{\rho ((\sigma - (\sigma - 1) (1 - \mu_j)) \rho + (\sigma - 1) (1 - \eta))^2} < 0,$$

which are the two key novel comparative statics highlighted in the main text of that section. Again, this last cross-partial derivative is useful in deriving predictions for the offshoring share  $\Upsilon_O$  since for  $\mu_{hN} \geq \mu_{hS}$  and  $\mu_{mN} \geq \mu_{mS}$ , this result implies  $\frac{\partial \ln(\Gamma_O/\Gamma_D)}{\partial \rho} = \frac{\partial \ln(\Gamma_O)}{\partial \rho} - \frac{\partial \ln(\Gamma_D)}{\partial \rho} \geq 0$ . In sum, whenever contract enforcement is higher in domestic transactions relative to offshore transactions, higher degrees of input substitutability tend to be associated with higher offshoring shares  $\Upsilon_O$ .

## A.3 Derivation of Some Results in Chapter 6

### Intrafirm Trade Shares with an Alternative Ranking of Fixed Costs

In Chapter 6, I computed intrafirm trade shares under the assumption that the ranking of fixed costs is given by  $f_{OV} > f_{OO} > f_{DV} > f_{DO}$ . This is a standard assumption in the literature (see for instance, Antràs and Helpman, 2004, 2008). Nevertheless, the evidence from Spain discussed in Chapter 8 suggests that perhaps a more empirically plausible ranking of fixed costs is as follows:

$$f_{OV} > f_{DV} > f_{OO} > f_{DO}.$$

In this Appendix I study the robustness of the results to assuming this alternative ranking of fixed costs. As already mentioned in Chapter 6, the share  $Sh_{i-f}$  of intrafirm imported inputs over the total imported input purchases in this case is given by

$$Sh_{i-f} = \frac{\lambda^{1-\sigma} \int_{\tilde{\varphi}_{OV}}^{\infty} \varphi^{\sigma-1} dG(\varphi)}{\Gamma_{OO} \int_{\tilde{\varphi}_{OO}}^{\tilde{\varphi}_{DV}} \varphi^{\sigma-1} dG(\varphi) + \lambda^{1-\sigma} \int_{\tilde{\varphi}_{OV}}^{\infty} \varphi^{\sigma-1} dG(\varphi)},$$

which, assuming a Pareto distribution of productivity with shape parameter  $\kappa > \sigma - 1$ , reduces to

$$Sh_{i-f} = \frac{\lambda^{1-\sigma}}{\Gamma_{OO} \left[ \left( \frac{\tilde{\varphi}_{OV}}{\tilde{\varphi}_{OO}} \right)^{\kappa-\sigma-1} - \left( \frac{\tilde{\varphi}_{OV}}{\tilde{\varphi}_{DV}} \right)^{\kappa-\sigma-1} \right] + \lambda^{1-\sigma}}.$$

Given the sorting in Figure 6.6, the key ratios of thresholds in the above equation satisfy:

$$\begin{aligned} \frac{\tilde{\varphi}_{OV}}{\tilde{\varphi}_{OO}} &= \left[ \frac{f_{OV} - f_{DV}}{f_{OO} - f_{DO}} \times \frac{\Gamma_{OO}}{\lambda^{1-\sigma}} \times \frac{1 - (w_N/\tau w_S)^{-(1-\eta)(\sigma-1)} \Gamma_{DO}/\Gamma_{OO}}{1 - (w_N/\tau w_S)^{-(1-\eta)(\sigma-1)}} \right]^{1/(\sigma-1)} ; \\ \frac{\tilde{\varphi}_{OV}}{\tilde{\varphi}_{DV}} &= \left[ \frac{f_{OV} - f_{DV}}{f_{DV} - f_{OO}} \times \frac{\lambda^{1-\sigma} (w_N/\tau w_S)^{-(1-\eta)(\sigma-1)} - \Gamma_{OO}}{\lambda^{1-\sigma} - \lambda^{1-\sigma} (w_N/\tau w_S)^{-(1-\eta)(\sigma-1)}} \right]^{1/(\sigma-1)}. \end{aligned}$$

Notice that  $\tilde{\varphi}_{OV}/\tilde{\varphi}_{OO}$  increases in  $\Gamma_{OO}$  and  $\lambda$ , and decreases in  $\Gamma_{DO}/\Gamma_{OO}$  and  $(w_N/\tau w_S)^{-(1-\eta)(\sigma-1)}$  (for the natural case in which  $\Gamma_{OO} < \Gamma_{DO}$ ). Conversely,  $\tilde{\varphi}_{OV}/\tilde{\varphi}_{DV}$  decreases in  $\Gamma_{OO}$  and  $\lambda$ , and increases in  $(w_N/\tau w_S)^{-(1-\eta)(\sigma-1)}$ . We can thus conclude that  $Sh_{i-f}$  necessarily decreases in  $\Gamma_{OO}$  and  $\lambda$ , and increases in  $(w_N/\tau w_S)^{-(1-\eta)(\sigma-1)}$  and  $\Gamma_{DO}/\Gamma_{OO}$ .

Invoking the comparative statics derived for the transaction-cost model (see, for instance, Table 5.5) we can thus conclude that  $Sh_{i-f}$  is shaped by parameter values in the same manner as in the case with the fixed costs ranked according to  $f_{OV} > f_{OO} > f_{DV} > f_{DO}$ , and thus:

$$Sh_{i-f} = Sh_{i-f} \left( \lambda, w_N/w_S, \tau, \kappa, \phi, \mu_S, \epsilon, \rho, \sigma, \eta \right).$$

Let us next consider how the implications of the property-rights model are affected by this alternative ranking of fixed costs and implied sorting pattern consistent with the Spanish data. In such a case, the intrafirm trade share is given by

$$Sh_{i-f} = \frac{\Gamma_{OV}/\Gamma_{OO}}{\left[ \left( \frac{\tilde{\varphi}_{OV}}{\tilde{\varphi}_{OO}} \right)^{\kappa-\sigma-1} - \left( \frac{\tilde{\varphi}_{OV}}{\tilde{\varphi}_{DV}} \right)^{\kappa-\sigma-1} \right] + \Gamma_{OV}/\Gamma_{OO}},$$

with

$$\begin{aligned} \frac{\tilde{\varphi}_{OV}}{\tilde{\varphi}_{OO}} &= \left[ \frac{f_{OV} - f_{DV}}{f_{OO} - f_{DO}} \times \frac{1 - (w_N/\tau w_S)^{-(1-\eta)(\sigma-1)} \Gamma_{DO}/\Gamma_{OO}}{\Gamma_{OV}/\Gamma_{OO} - (w_N/\tau w_S)^{-(1-\eta)(\sigma-1)} \Gamma_{DV}/\Gamma_{OO}} \right]^{1/(\sigma-1)} ; \\ \frac{\tilde{\varphi}_{OV}}{\tilde{\varphi}_{DV}} &= \left[ \frac{f_{OV} - f_{DV}}{f_{DV} - f_{OO}} \times \frac{(w_N/\tau w_S)^{-(1-\eta)(\sigma-1)} \Gamma_{DV}/\Gamma_{OO} - 1}{\Gamma_{OV}/\Gamma_{OO} - (w_N/\tau w_S)^{-(1-\eta)(\sigma-1)} \Gamma_{DV}/\Gamma_{OO}} \right]^{1/(\sigma-1)}. \end{aligned}$$

It is clear that the intrafirm trade share thus continues to be increasing in the key ratio  $\Gamma_{OV}/\Gamma_{OO}$ . Furthermore,  $Sh_{i-f}$  continues to be increasing in  $\Gamma_{DO}/\Gamma_{OO}$  reflecting a selection into offshore outsourcing effect.

An important novel feature of these equations is that they now also depend on the ratio  $\Gamma_{DV}/\Gamma_{OO}$ , and the overall dependence of the share of intrafirm trade on this term is ambiguous. Whenever  $\Gamma_{OV} = \Gamma_{DV}$ , the equations for  $\tilde{\varphi}_{OV}/\tilde{\varphi}_{OO}$  and  $\tilde{\varphi}_{OV}/\tilde{\varphi}_{DV}$  simplify significantly, and one can show that the intrafirm trade share continues to be increasing in  $\Gamma_{OV}/\Gamma_{OO}$  and  $\Gamma_{DO}/\Gamma_{OO}$  as in the case of our Benchmark model. Nevertheless, in the presence of differences in contractibility between domestic and foreign sourcing, we would expect that  $\Gamma_{OV} < \Gamma_{DV}$ , hence complicating matters.

Another point worth noting is that the overall effect of  $(w_N/\tau w_S)^{-(1-\eta)(\sigma-1)}$  is no longer unambiguous because although  $\tilde{\varphi}_{OV}/\tilde{\varphi}_{DV}$  is clearly increasing in this term, the effect of  $(w_N/\tau w_S)^{-(1-\eta)(\sigma-1)}$  on  $\tilde{\varphi}_{OV}/\tilde{\varphi}_{OO}$  crucially depends on the relative size of  $\Gamma_{OV}/\Gamma_{OO}$  and  $\Gamma_{DV}/\Gamma_{DO}$ . In particular, when  $\Gamma_{OV}/\Gamma_{OO} < \Gamma_{DV}/\Gamma_{DO}$ , we now have that  $\tilde{\varphi}_{OV}/\tilde{\varphi}_{OO}$  is increasing in  $(w_N/\tau w_S)^{-(1-\eta)(\sigma-1)}$ , and thus through this mechanism, the intrafirm trade share might be negatively correlated with trade frictions  $\tau$ , consistently with our regression results in Chapter 8. Conversely, when  $\Gamma_{OV}/\Gamma_{OO} > \Gamma_{DV}/\Gamma_{DO}$ , we revert to a scenario analogous to the transaction-cost model, in which  $\tilde{\varphi}_{OV}/\tilde{\varphi}_{OO}$  is decreasing in  $(w_N/\tau w_S)^{-(1-\eta)(\sigma-1)}$  and the overall effect of  $\tau$  on the intrafirm trade share is necessarily positive. Which of the inequalities  $\Gamma_{OV}/\Gamma_{OO} < \Gamma_{DV}/\Gamma_{DO}$  or  $\Gamma_{OV}/\Gamma_{OO} > \Gamma_{DV}/\Gamma_{DO}$  is more reasonable? Building on the insights from our extension with partial contractibility, we can conclude that if the higher contractual insecurity of offshoring stems largely from lower contractibility of headquarter services, we will have that  $\Gamma_{OV}/\Gamma_{OO} > \Gamma_{DV}/\Gamma_{DO}$ . Conversely, when lower contractibility of input manufacturing is the key source of contractual insecurity in offshoring, we will instead have  $\Gamma_{OV}/\Gamma_{OO} < \Gamma_{DV}/\Gamma_{DO}$ .

In summary, with this alternative ranking of fixed costs, the comparative statics of the transaction-cost model are the same as with the Benchmark model sorting, while those of the property-rights model are now more complicated. This is due to the fact that the extensive margin of offshoring and foreign integration are shaped by the movements of several thresholds. Still the ratio  $\Gamma_{OV}/\Gamma_{OO}$  continues to be a key determinant of the intrafirm trade share. And the richer extensive margin effects imply that one cannot reject the property-rights model based on the negative effect of trade costs on that ratio estimated in Chapter 8.

## Downstreamness and Integration

In this section of the Appendix, I provide some more details on the variant of the transaction-cost model in Chapter 6 with sequential production. The analysis is similar to the one in the property-rights model in Antràs and Chor (2013).

As in Chapter 4, I focus on the case in which production does not use head-quarter services and offshore outsourcing is associated with totally incomplete contracts. As discussed in Chapter 4, the contractual efficiency of offshore outsourcing, given unconstrained ex-ante transfers and bargaining share  $\beta(v)$  for each state  $v$ , is given by equation (4.30), which I reproduce here:

$$\Gamma_\ell(\{\beta(v)\}_{v=0}^1) = \frac{(\sigma-1)}{(\sigma_\rho-1)} \left(\frac{\sigma_\rho}{\sigma}\right)^{\frac{\sigma-\sigma_\rho}{\sigma_\rho-1}} \int_0^1 \left\{ \left(\frac{\sigma_\rho}{1-\beta(v)} - (\sigma_\rho-1)\right) \times (1-\beta(v))^{\sigma_\rho} \left[ \int_0^v (1-\beta(u))^{\sigma_\rho-1} du \right]^{\frac{\sigma-\sigma_\rho}{\sigma_\rho-1}} \right\} dv. \quad (\text{A.12})$$

As discussed in Chapter 4, one can then solve for the optimal path of bargaining shares  $\beta^*(v)$  and see how it relates to  $v$  as a function of the other parameters of the model. The main lessons we obtained in that chapter is that when  $\sigma > \sigma_\rho$ ,  $\beta^*(v)$  is increasing in  $v$ , while when  $\sigma < \sigma_\rho$ ,  $\beta^*(v)$  is decreasing in  $v$ .

As discussed in Chapter 7, Antràs and Chor (2013) consider a situation where the firm cannot freely choose any arbitrary  $\beta(v)$  at any stage  $v$ , but rather has to decide whether or not to integrate the different suppliers, with integration being associated with a higher bargaining share ( $\beta_V$ ) than outsourcing ( $\beta_O$ ). Their results are explained in Chapter 7 and their derivations can be obtained from their paper and their Supplementary Appendix.

Consider instead a transaction-cost version of the model in which integration is not associated with higher bargaining power in the same contracting environment, but is rather associated with the ability to circumvent contracting and bargaining at the cost of some ‘governance costs’. More specifically, assume that when a supplier is owned by the final-good producer, the firm has the authority to force the supplier to choose a level of investment at stage  $v$  that maximizes its incremental contribution to revenue minus the (inflated) cost of investment provision. More formally, I assume that, under integration,  $m(v)$  is set to maximize  $\Delta R(v) - \lambda c_j m(v)$  rather than  $(1 - \beta_O)\Delta R(v) - c_j m(v)$ , where  $\Delta R(v)$  is given in equation (4.29). Thus integration resolves the hold-up problem at stage  $v$  but it is associated with higher governance costs (since  $\lambda > 1$ ).

The key question is then: in which type of stages is it crucial to resolve the hold-up problem? Our results on the optimal bargaining shares  $\beta^*(v)$  suggests that resolving the hold up problem via integration is particularly beneficial in upstream stages for  $\sigma > \sigma_\rho$ , and in downstream stages for  $\sigma < \sigma_\rho$ . In other words, downstreamness should have a negative effect on foreign integration relative to offshore outsourcing whenever inputs are sequential complements ( $\sigma > \sigma_\rho$ ), while it should have a positive effect on foreign integration when inputs are sequential substitutes ( $\sigma < \sigma_\rho$ ).

This result can be formalized along the lines of the proof of Proposition 2 in Antràs and Chor (2013). Take the case  $\sigma > \sigma_\rho$ , and suppose there exists a

stage  $\tilde{v} \in (0, 1)$  and a positive constant  $\varepsilon > 0$  such that stages in  $(\tilde{v} - \varepsilon, \tilde{v})$  are outsourced, while stages in  $(\tilde{v}, \tilde{v} + \varepsilon)$  are integrated. This situation would provide a counterexample of our claim that only the most upstream stages can possibly be integrated. We shall then show that this counterexample leads to a contradiction. Let the firm profits associated with this scenario be denoted by  $\Pi_1$ . On the other hand, consider an alternative organizational mode which instead integrates the stages in  $(\tilde{v} - \varepsilon, \tilde{v})$  and outsources the stages in  $(\tilde{v}, \tilde{v} + \varepsilon)$ , while retaining the same organizational decision for all other stages. Let profits from this alternative be  $\Pi_2$ . Both of these profit flows are naturally proportional to the indices of contractual efficiency  $\Gamma_\ell$  associated with each of these scenarios.

Using the expression for  $\Gamma_\ell$  in (A.12), one can then show that, up to a positive multiplicative constant:

$$\begin{aligned} \Pi_1 - \Pi_2 \propto & \int_{\tilde{v}-\varepsilon}^{\tilde{v}} \left( 1 - \frac{(\sigma_\rho - 1)}{\sigma_\rho} (1 - \beta_O) \right) (1 - \beta_O)^{\sigma_\rho - 1} [B + \varepsilon \lambda^{1 - \sigma_\rho} + (j - \tilde{v})(1 - \beta_O)^{\sigma_\rho - 1}]^{\frac{\sigma - \sigma_\rho}{\sigma_\rho - 1}} dj \\ & + \int_{\tilde{v}}^{\tilde{v} + \varepsilon} \frac{1}{\sigma_\rho} \lambda^{1 - \sigma_\rho} [B + (j - \tilde{v} + \varepsilon) \lambda^{1 - \sigma_\rho}]^{\frac{\sigma - \sigma_\rho}{\sigma_\rho - 1}} dj \\ & - \int_{\tilde{v}-\varepsilon}^{\tilde{v}} \frac{1}{\sigma_\rho} \lambda^{1 - \sigma_\rho} [B + \varepsilon (1 - \beta_O)^{\sigma_\rho - 1} + (j - \tilde{v}) \lambda^{1 - \sigma_\rho}]^{\frac{\sigma - \sigma_\rho}{\sigma_\rho - 1}} dj \\ & - \int_{\tilde{v}}^{\tilde{v} + \varepsilon} \left( 1 - \frac{(\sigma_\rho - 1)}{\sigma_\rho} (1 - \beta_O) \right) (1 - \beta_O)^{\sigma_\rho - 1} [B + (j - \tilde{v} + \varepsilon)(1 - \beta_O)^{\sigma_\rho - 1}]^{\frac{\sigma - \sigma_\rho}{\sigma_\rho - 1}} dj. \end{aligned}$$

where we define  $B \equiv \int_0^{\tilde{v} - \varepsilon} \lambda^{1 - \sigma_\rho} dk$  (since those upstream stages are integrated given that  $1 - \beta^*(0) \rightarrow +\infty$ ). That the difference in profits depends only on profits in the interval  $(\tilde{v} - \varepsilon, \tilde{v} + \varepsilon)$  and is not affected by decisions downstream follows from the fact that we have chosen the width  $\varepsilon$  to be common for both sub-intervals. Evaluating the integrals above with respect to  $j$  and simplifying, we obtain after some tedious algebra:

$$\begin{aligned} \Pi_1 - \Pi_2 \propto & \beta_O \frac{\sigma_\rho - 1}{\sigma_\rho} \left[ (B + \varepsilon \lambda^{1 - \sigma_\rho})^{\frac{\sigma - \sigma_\rho}{\sigma_\rho - 1}} + (B + \varepsilon (1 - \beta_O)^{\sigma_\rho - 1})^{\frac{\sigma - \sigma_\rho}{\sigma_\rho - 1}} \right. \\ & \left. - (B + \varepsilon \lambda^{1 - \sigma_\rho} + \varepsilon (1 - \beta_O)^{\sigma_\rho - 1})^{\frac{\sigma - \sigma_\rho}{\sigma_\rho - 1}} - B^{\frac{\sigma - \sigma_\rho}{\sigma_\rho - 1}} \right]. \end{aligned}$$

To show a contradiction, i.e.,  $\Pi_1 - \Pi_2 < 0$ , it thus suffices to show that the expression in square parentheses is negative. To see this, consider the function  $f(y) = y^{\frac{\sigma - \sigma_\rho}{\sigma_\rho - 1}}$ . Simple differentiation will show that for  $y, a > 0$  and  $b \geq 0$ ,  $f(y + a + b) - f(y + b)$  is an increasing function in  $b$  when  $\sigma > \sigma_\rho$ . Hence,  $(y + a + b)^{\frac{\sigma - \sigma_\rho}{\sigma_\rho - 1}} - (y + b)^{\frac{\sigma - \sigma_\rho}{\sigma_\rho - 1}} > (y + a)^{\frac{\sigma - \sigma_\rho}{\sigma_\rho - 1}} - (y)^{\frac{\sigma - \sigma_\rho}{\sigma_\rho - 1}}$ . Setting  $y = B$ ,  $a = \varepsilon (1 - \beta_O)^{\sigma_\rho - 1}$  and  $b = \varepsilon \lambda^{1 - \sigma_\rho}$ , it follows that the last term in square brackets is negative and that  $\Pi_1 - \Pi_2 < 0$ . This yields the desired contradiction as profits can

be strictly increased by switching to the organizational mode that yields profits  $\Pi_2$ . The proof that integration will occur in the most downstream stages whenever  $\sigma < \sigma_\rho$  can be established using an analogous proof by contradiction.

## A.4 The Determinants of the Ratio $\Gamma_{OV}/\Gamma_{OO}$ in Chapter 7

### Basic Model

We will first prove that the ratio  $\Gamma_{OV}/\Gamma_{OO}$  in the basic model – see equation (7.6) – is monotonically increasing in  $\eta$ . To be able to apply this same proof to environments with partial contractibility, I begin by writing (7.6) in a slighter more general form:

$$\frac{\Gamma_{OV}}{\Gamma_{OO}} = \left( \frac{1 - \beta_V \omega_h - (1 - \beta_V) \omega_m}{1 - \beta_O \omega_h - (1 - \beta_O) \omega_m} \right)^{\sigma(1 - \omega_h - \omega_m)} \left( \frac{\beta_V}{\beta_O} \right)^{\sigma \omega_h} \left( \frac{1 - \beta_V}{1 - \beta_O} \right)^{\sigma \omega_m}. \quad (\text{A.13})$$

To obtain (7.6) from (A.13) one simply needs to set  $\omega_h = (\sigma - 1)\eta/\sigma$  and  $\omega_m = (\sigma - 1)(1 - \eta)/\sigma$ .

I will next show that the ratio  $\Gamma_{OV}/\Gamma_{OO}$  in (A.13) is monotonically increasing in  $\omega_h$  and monotonically decreasing in  $\omega_m$ . It is clear that this will imply, in turn, that this ratio is increasing in  $\eta$ . The proof builds on, but greatly simplifies, the one in Antràs and Helpman (2008).

Let us start with the effect of  $\omega_m$ . Straightforward differentiation of the log of the ratio  $\Gamma_{OV}/\Gamma_{OO}$  in (A.13) delivers

$$\begin{aligned} \frac{1}{\sigma} \frac{\partial \ln(\Gamma_{OV}/\Gamma_{OO})}{\partial \omega_m} &= \ln \left( \frac{1 - \beta_V}{1 - \beta_O} \right) - \ln \left( \frac{1 - \beta_V \omega_h - (1 - \beta_V) \omega_m}{1 - \beta_O \omega_h - (1 - \beta_O) \omega_m} \right) \\ &\quad + \frac{(1 - \omega_h)(1 - \omega_h - \omega_m)(\beta_V - \beta_O)}{(1 - \beta_O \omega_h - (1 - \beta_O) \omega_m)(1 - \beta_V \omega_h - (1 - \beta_V) \omega_m)}. \end{aligned} \quad (\text{A.14})$$

We next further differentiate with respect to  $\beta_O$  to obtain

$$\frac{1}{\sigma} \frac{\partial^2 \ln(\Gamma_{OV}/\Gamma_{OO})}{\partial \omega_m \partial \beta_O} = (1 - \omega_h) \frac{\beta_O(1 - \omega_h) + \omega_h(1 - \beta_O)}{(1 - \beta_O)(1 - \beta_O \omega_h - (1 - \beta_O) \omega_m)^2} > 0.$$

Because  $\beta_V \geq \beta_O$ , the largest possible value that  $\partial \ln(\Gamma_{OV}/\Gamma_{OO})/\partial \omega_m$  can take is when evaluated at  $\beta_O = \beta_V$ . But in such a case, we have that  $\partial \ln(\Gamma_{OV}/\Gamma_{OO})/\partial \omega_m$  equals 0. It then follows that for any  $\beta_O < \beta_V$ , we must have  $\partial \ln(\Gamma_{OV}/\Gamma_{OO})/\partial \omega_m < 0$ . Hence,  $\Gamma_{OV}/\Gamma_{OO}$  is monotonically decreasing in  $\omega_m$ .

The proof that  $\partial \ln(\Gamma_{OV}/\Gamma_{OO})/\partial \omega_h > 0$  can be proved analogously. It suffices to note that letting  $\beta_{mV} = 1 - \beta_V$  and  $\beta_{mO} = 1 - \beta_O$ , we can write (A.13) as

$$\frac{\Gamma_{OV}}{\Gamma_{OO}} = \left( \frac{1 - \beta_{mO} \omega_m - (1 - \beta_{mO}) \omega_h}{1 - \beta_{mV} \omega_m - (1 - \beta_{mV}) \omega_h} \right)^{-\sigma(1 - \omega_h - \omega_m)} \left( \frac{\beta_{mO}}{\beta_{mV}} \right)^{-\sigma \omega_m} \left( \frac{1 - \beta_{mO}}{1 - \beta_{mV}} \right)^{-\sigma \omega_h}.$$

Importantly, we now have  $\beta_{mO} > \beta_{mV}$ , and thus this expression is isomorphic to (A.13) above except for the negative exponents. We can thus conclude that if  $\Gamma_{OV}/\Gamma_{OO}$  is monotonically decreasing in  $\omega_m$ , then it must be monotonically increasing in  $\omega_h$ .

In the transaction-cost model we showed that  $\Gamma_{OO}$  was decreasing in the elasticity of demand  $\sigma$ , and thus on this account the relative attractiveness of integration was increasing in this parameter. In this property-rights model, the effect of  $\sigma$  on the ratio  $\Gamma_{OV}/\Gamma_{OO}$  is complex and depends non-monotonically on the other parameters of the model. Coupled with the various effects of  $\sigma$  on the other determinants of the share of intrafirm trade, the overall effect of this parameter is ambiguous.

### Financial Constraints

We next turn to the model with financial constraints, in which the ratio  $\Gamma_{OV}/\Gamma_{OO}$  is given in equation (7.13). For reasons that will become clear, I rewrite this expression as

$$\frac{\Gamma_{OV}}{\Gamma_{OO}} = \left( \frac{\beta_V(1-\omega_h) + \phi(1-\beta_V)(1-\omega_m)}{\beta_O(1-\omega_h) + \phi(1-\beta_O)(1-\omega_m)} \right)^{\sigma(1-\omega_h-\omega_m)} \left( \frac{\beta_V}{\beta_O} \right)^{\sigma\omega_h} \left( \frac{1-\beta_V}{1-\beta_O} \right)^{\sigma\omega_m}, \quad (\text{A.15})$$

where  $\omega_h = (\sigma - 1)\eta/\sigma$  and  $\omega_m = (\sigma - 1)(1 - \eta)/\sigma$ .

We first demonstrate the claim in the main text that the incentive to integrate suppliers is higher, the tighter are financial constraints, in the sense that  $\Gamma_{OV}/\Gamma_{OO}$  is decreasing in  $\phi$ . This follows from simple differentiation:

$$\frac{1}{\sigma} \frac{\partial \ln(\Gamma_{OV}/\Gamma_{OO})}{\partial \phi} = \frac{-(1-\omega_h)(1-\omega_m)(1-\omega_h-\omega_m)(\beta_V-\beta_O)}{(\beta_O(1-\omega_h) + \phi(1-\beta_O)(1-\omega_m))(\beta_V(1-\omega_h) + \phi(1-\beta_V)(1-\omega_m))} < 0. \quad (\text{A.16})$$

We now show that, even in the presence of financial constraints, the ratio  $\ln(\Gamma_{OV}/\Gamma_{OO})$  continues to be monotonic in  $\eta$ . The proof is very closely related to the one developed above for the basic model. In particular, we first take logs and differentiate (7.13) to find

$$\begin{aligned} \frac{1}{\sigma} \frac{\partial \ln(\Gamma_{OV}/\Gamma_{OO})}{\partial \omega_m} &= \ln \left( \frac{1-\beta_V}{1-\beta_O} \right) - \ln \left( \frac{\beta_V(1-\omega_h) + \phi(1-\beta_V)(1-\omega_m)}{\beta_O(1-\omega_h) + \phi(1-\beta_O)(1-\omega_m)} \right) \\ &+ \frac{\phi(1-\omega_h)(1-\omega_h-\omega_m)(\beta_V-\beta_O)}{(\beta_O(1-\omega_h) + \phi(1-\beta_O)(1-\omega_m))(\beta_V(1-\omega_h) + \phi(1-\beta_V)(1-\omega_m))}, \end{aligned}$$

which again collapses to 0 when  $\beta_O = \beta_V$ . To complete the proof, it then suffices to note that

$$\frac{1}{\sigma} \frac{\partial^2 \ln(\Gamma_{OV}/\Gamma_{OO})}{\partial \omega_m \partial \beta_O} = (1-\omega_h) \frac{\beta_O(1-\omega_h) + \phi\omega_h(1-\beta_O)}{(1-\beta_O)(\beta_O(1-\omega_h) + \phi(1-\beta_O)(1-\omega_m))^2} > 0,$$

and thus  $\partial \ln(\Gamma_{OV}/\Gamma_{OO})/\partial\omega_m < 0$  for any  $\beta_O < \beta_V$ . The proof that  $\partial \ln(\Gamma_{OV}/\Gamma_{OO})/\partial\omega_h > 0$  is entirely analogous and is omitted to save space.

Next, we also note that straightforward differentiation of the expression for  $\partial \ln(\Gamma_{OV}/\Gamma_{OO})/\partial\phi$  in (A.16) also demonstrates that

$$\frac{\partial^2 \ln(\Gamma_{OV}/\Gamma_{OO})}{\partial\phi\partial\omega_h} > 0 \text{ and } \frac{\partial^2 \ln(\Gamma_{OV}/\Gamma_{OO})}{\partial\phi\partial\omega_m} > 0, \quad (\text{A.17})$$

and thus the positive effect of financial constraints on the attractiveness of integration is lower, the higher are  $\omega_h$  and  $\omega_m$ .<sup>26</sup> These results imply that, unlike in our basic transaction-cost model, in our property-rights model it no longer is the case that an increase in improvement in the quality of financial contracting (higher  $\phi$ ) will have a differentially large positive effect on the profitability of outsourcing in production processes with high headquarter intensity (i.e.,  $\partial(\partial(\Gamma_{OV}/\Gamma_{OO})/\partial\phi)/\partial\eta < 0$ ). In particular, since  $\eta$  shapes  $\omega_h$  and  $\omega_m$  in opposite directions, it is not hard to find numerical examples in which  $\partial(\partial(\Gamma_{OV}/\Gamma_{OO})/\partial\phi)/\partial\eta > 0$ . Thus, our property-rights model does not have clear predictions for the effects of interactions of empirical proxies for  $\phi$  and  $\eta$  on the share of intrafirm trade.

### Partial Contractibility

Consider now the variant of the model with partial contractibility in international transactions, and let the degree of contractibility vary across inputs and countries. As mentioned in the main text, the ratio  $\Gamma_{OV}/\Gamma_{OO}$  in such a case is given by equation (7.14). But note that defining  $\omega_h = (\sigma - 1)\eta(1 - \mu_{hS})/\sigma$  and  $\omega_m = (\sigma - 1)(1 - \eta)(1 - \mu_{mS})/\sigma$ , we can express (7.14) as

$$\frac{\Gamma_{OV}}{\Gamma_{OO}} = \left( \frac{1 - \beta_V\omega_h - (1 - \beta_V)\omega_m}{1 - \beta_O\omega_h - (1 - \beta_O)\omega_m} \right)^{\sigma(1 - \omega_h - \omega_m)} \left( \frac{\beta_V}{\beta_O} \right)^{\sigma\omega_h} \left( \frac{1 - \beta_V}{1 - \beta_O} \right)^{\sigma\omega_m}.$$

It should be clear that this expression is identical to equation (A.13), which we studied earlier in our discussion of the comparative statics in the Basic Model. We can thus refer to our earlier results to confirm that  $\Gamma_{OV}/\Gamma_{OO}$  is decreasing in the contractibility of headquarter services  $\mu_{hS}$  and increasing in the contractibility of manufacturing  $\mu_{mS}$ . It is also clear that the ratio  $\Gamma_{OV}/\Gamma_{OO}$  continues to be increasing in headquarter intensity  $\eta$  for *any* level of contractibility. Finally, from our previous analysis, we can also state that the effect of  $\sigma$  on the ratio  $\Gamma_{OV}/\Gamma_{OO}$  is ambiguous.

We next discuss how changes in the level of contractibility shape differentially the ratio  $\Gamma_{OV}/\Gamma_{OO}$  depending on other characteristics of production. A first result follows immediately from the definition of  $\omega_h$  and  $\omega_m$ . In particular, the negative effect of  $\mu_{hS}$  on  $\Gamma_{OV}/\Gamma_{OO}$  is magnified by high levels of  $\eta$ ,

<sup>26</sup>More specifically, in each case,  $\partial \ln(\Gamma_{OV}/\Gamma_{OO})/\partial\phi$  can be decomposed as the product of two ratios that are each increasing in  $\omega_h$  or  $\omega_m$ .

while the positive effect of  $\mu_{mS}$  on  $\Gamma_{OV}/\Gamma_{OO}$  is decreasing in  $\eta$ . Or, more formally,  $\partial(\partial \ln(\Gamma_{OV}/\Gamma_{OO})/\partial \mu_{hS})/\partial \eta < 0$  and  $\partial(\partial \ln(\Gamma_{OV}/\Gamma_{OO})/\partial \mu_{mS})/\partial \eta < 0$ . A second result follows from the same definitions of  $\omega_h$  and  $\omega_m$ , and the fact that the effects of contractibility are always enhanced by a large  $\sigma$ . In particular, note that in equation (A.13), contractibility shows up in the following terms:  $\omega_h$ ,  $\omega_m$ ,  $\sigma\omega_h = (\sigma - 1)\eta(1 - \mu_{hS})$ ,  $\sigma\omega_m = (\sigma - 1)(1 - \eta)(1 - \mu_{mS})$ , and  $\sigma(1 - \omega_h - \omega_m) = 1 + (\sigma - 1)\eta\mu_{hS} + (\sigma - 1)(1 - \eta)\mu_{mS}$ . We can thus conclude that the effect of  $\mu_{hS}$  and  $\mu_{mS}$  are necessarily attenuated by a low  $\sigma$ , which implies that  $\partial(\partial \ln(\Gamma_{OV}/\Gamma_{OO})/\partial \mu_{hS})/\partial \sigma < 0$  and  $\partial(\partial \ln(\Gamma_{OV}/\Gamma_{OO})/\partial \mu_{mS})/\partial \sigma > 0$ .

### Partial Relationship-Specificity

As argued in the main text, in the extension of the model with partial relationship-specificity, it becomes harder to obtain sharp comparative statics. Let us consider one case in which we do find an analytical result. Suppose Nash bargaining is symmetric, so  $\beta_O = 1/2$  and  $\beta_V = (1 + \delta)/2$ . Defining  $\omega_h = (\sigma - 1)\eta(1 - \mu_{hS})/\sigma$  and  $\omega_m = (\sigma - 1)(1 - \eta)(1 - \mu_{mS})/\sigma$ , and allowing for a distinct specificity for headquarter services ( $\epsilon_h$ ) and for the manufacturing input ( $\epsilon_m$ ), we find that the ratio  $\Gamma_{OV}/\Gamma_{OO}$  in (7.16) can be expressed as:

$$\frac{\Gamma_{OV}}{\Gamma_{OO}} = \left( \frac{1 - (1 - \frac{1}{2}(1 - \delta)\epsilon_h)\omega_h - (1 - \frac{1}{2}(1 + \delta)\epsilon_m)\omega_m}{1 - (1 - \frac{1}{2}\epsilon_h)\omega_h - (1 - \frac{1}{2}\epsilon_m)\omega_m} \right)^{\sigma(1 - \omega_h - \omega_m)} \\ \times \left( \frac{1 - \frac{1}{2}(1 - \delta)\epsilon_h}{1 - \frac{1}{2}\epsilon_h} \right)^{\sigma\omega_h} \left( \frac{1 - \frac{1}{2}(1 + \delta)\epsilon_m}{1 - \frac{1}{2}\epsilon_m} \right)^{\sigma\omega_m}.$$

Straightforward differentiation delivers

$$\frac{1}{\sigma} \frac{\partial \ln(\Gamma_{OV}/\Gamma_{OO})}{\partial \epsilon_h} = \frac{1}{2} \delta \frac{\omega_h}{(1 - \epsilon_h + \frac{1}{2}(1 + \delta)\epsilon_h)(1 - \epsilon_h + \frac{1}{2}\epsilon_h)} \\ - \frac{1}{2} \delta \frac{\omega_h(1 - \omega_h - \omega_m)(1 - \omega_h - \omega_m + \epsilon_m\omega_m)}{(1 - (1 - \frac{1}{2}\epsilon_h)\omega_h - (1 - \frac{1}{2}\epsilon_m)\omega_m)(1 - (1 - \frac{1}{2}(1 - \delta)\epsilon_h)\omega_h - (1 - \frac{1}{2}(1 + \delta)\epsilon_m)\omega_m)}.$$

It is clear that this derivative is increasing in  $\epsilon_h$ , and thus it cannot be lower than when evaluated at  $\epsilon_h = 0$ . And in that case, we have

$$\frac{1}{\sigma} \frac{\partial \ln(\Gamma_{OV}/\Gamma_{OO})}{\partial \epsilon_h} \Big|_{\epsilon_h=0} = \frac{\frac{1}{8}\delta\omega_h\omega_m\epsilon_m(2\delta(1 - \omega_h - \omega_m) + \omega_m\epsilon_m(1 + \delta))}{(1 - \omega_h - (1 - \frac{1}{2}\epsilon_m)\omega_m)(1 - \omega_h - (1 - \frac{1}{2}(1 + \delta)\epsilon_m)\omega_m)} > 0.$$

This confirms that the ratio  $\Gamma_{OV}/\Gamma_{OO}$  is increasing in  $\epsilon_h$  whenever Nash bargaining is symmetric and  $\beta_O = 1/2$ . When departing from this symmetric Nash bargaining assumption, the derivative  $\partial \ln(\Gamma_{OV}/\Gamma_{OO})/\partial \epsilon_h$  continues to be minimized at  $\epsilon_h = 0$ , but for a low value of  $\beta_O$  and  $\beta_V$  it may take a negative value at  $\epsilon_h = 0$ .

More specifically, the key condition is  $\beta_V \beta_O \epsilon_m \omega_m > (1 - \beta_O - \beta_V)(1 - \omega_h - \omega_m)$ , which is satisfied for  $\beta_O = 1/2$  and  $\beta_V = (1 + \delta)/2$  but not necessarily for sufficiently low values of these parameters.

Following analogous steps, it is also possible to show that the ratio  $\Gamma_{OV}/\Gamma_{OO}$  is typically decreasing in  $\epsilon_m$ , with the effect of this parameter being unambiguously negative whenever  $\beta_V = 1/2$ , and thus  $\beta_O = (1/2 - \delta)/(1 - \delta)$ .

## Multiple Inputs and Multilateral Contracting

As argued in the main text, the equilibrium of the extension with multiple inputs and multilateral contracting is analogous to that in the model with just one  $F$  and  $M$  agents, but with bargaining powers  $\beta_{hO}$ ,  $\beta_{mO}$ ,  $\beta_{hV}$ , and  $\beta_{mV}$  given by

$$\begin{aligned}\beta_{hO} &= \beta_{mO} = \frac{\sigma\rho}{(\sigma-1)(1-\eta) + \sigma\rho} \\ \beta_{hV} &= \frac{(\sigma-1)(1-\eta)\delta + \sigma\rho}{(\sigma-1)(1-\eta) + \sigma\rho} > \frac{\sigma\rho(1-\delta)}{(\sigma-1)(1-\eta) + \sigma\rho} = \beta_{mV}.\end{aligned}$$

We can then plug these values in a general formula analogous to (A.13) and given by

$$\frac{\Gamma_{OV}}{\Gamma_{OO}} = \left( \frac{1 - \beta_{hV}\omega_h - \beta_{mV}\omega_m}{1 - \beta_{hO}\omega_h - \beta_{mO}\omega_m} \right)^{\sigma(1-\omega_h-\omega_m)} \left( \frac{\beta_{hV}}{\beta_{hO}} \right)^{\sigma\omega_h} \left( \frac{\beta_{mV}}{\beta_{mO}} \right)^{\sigma\omega_m},$$

where again  $\omega_h = (\sigma-1)\eta(1-\mu_{hS})/\sigma$  and  $\omega_m = (\sigma-1)(1-\eta)(1-\mu_{mS})/\sigma$ . As claimed in the main text, this substitution results, after some manipulations, in:

$$\begin{aligned}\frac{\Gamma_{OV}}{\Gamma_{OO}} &= \left( 1 - \frac{(\sigma-1)(1-\eta)\delta\omega_h - \sigma\rho\delta\omega_m}{(\sigma-1)(1-\eta) + \sigma\rho(1-\omega_h-\omega_m)} \right)^{\sigma(1-\omega_h-\omega_m)} \\ &\quad \times \left( 1 + \frac{(\sigma-1)(1-\eta)\delta}{\sigma\rho} \right)^{\sigma\omega_h} (1-\delta)^{\sigma\omega_m}.\end{aligned}\tag{A.18}$$

We first prove that the ratio  $\Gamma_{OV}/\Gamma_{OO}$  in this expression continues to be increasing in  $\omega_h$  and decreasing in  $\omega_m$ . To see this, take logs of (A.18) and differentiate, to obtain:

$$\begin{aligned}\frac{1}{\sigma} \frac{\partial \ln(\Gamma_{OV}/\Gamma_{OO})}{\partial \omega_h} &= -\ln \left( 1 - \frac{(\sigma-1)(1-\eta)\delta\omega_h - \sigma\rho\delta\omega_m}{(\sigma-1)(1-\eta) + \sigma\rho(1-\omega_h-\omega_m)} \right) + \ln \left( 1 + \frac{(\sigma-1)(1-\eta)\delta}{\sigma\rho} \right) \\ &\quad - \frac{\delta((\sigma-1)(1-\eta) + \sigma\rho)((\sigma-1)(1-\eta) - \sigma\rho\omega_m)(1-\omega_h-\omega_m)}{((\sigma-1)(1-\eta) + \sigma\rho(1-\omega_h-\omega_m))^2 \left( 1 - \frac{(\sigma-1)(1-\eta)\delta\omega_h - \sigma\rho\delta\omega_m}{(\sigma-1)(1-\eta) + \sigma\rho(1-\omega_h-\omega_m)} \right)},\end{aligned}$$

where  $(\sigma - 1)(1 - \eta) - \sigma\rho\omega_m = (\sigma - 1)(1 - \eta)(1 - \rho(1 - \mu_{mS})) > 0$ . Next, notice that further differentiating with respect to  $\delta$ , we find:

$$\begin{aligned} \frac{1}{\sigma} \frac{\partial \ln(\Gamma_{OV}/\Gamma_{OO})}{\partial \omega_h \partial \delta} &= \frac{((\sigma - 1)(1 - \eta) + \sigma\rho)((\sigma - 1)(1 - \eta) - \sigma\rho\omega_m)}{((\sigma - 1)(1 - \eta) + \sigma\rho(1 - \omega_h - \omega_m))^2} \\ &\times \frac{((\sigma - 1)(1 - \eta)(1 - \delta + \delta\omega_m) + \sigma\delta\rho\omega_m)}{((\sigma - 1)\delta(1 - \eta) + \sigma\rho) \left(1 - \frac{(\sigma - 1)(1 - \eta)\delta\omega_h - \sigma\rho\delta\omega_m}{(\sigma - 1)(1 - \eta) + \sigma\rho(1 - \omega_h - \omega_m)}\right)^2} > 0. \end{aligned}$$

It thus follows that  $\partial \ln(\Gamma_{OV}/\Gamma_{OO})/\partial \omega_h$  cannot be lower than when evaluated at  $\delta = 0$ , at which it is clear from the above expression that this derivative is zero. In sum, we have  $\partial \ln(\Gamma_{OV}/\Gamma_{OO})/\partial \omega_h > 0$  for all  $\delta > 0$ .

Next, differentiation delivers

$$\begin{aligned} \frac{1}{\sigma} \frac{\partial \ln(\Gamma_{OV}/\Gamma_{OO})}{\partial \omega_m} &= -\ln\left(1 - \frac{(\sigma - 1)(1 - \eta)\delta\omega_h - \sigma\rho\delta\omega_m}{(\sigma - 1)(1 - \eta) + \sigma\rho(1 - \omega_h - \omega_m)}\right) + \ln(1 - \delta) \\ &+ \frac{(1 - \omega_h - \omega_m)(1 - \omega_h)\sigma\delta\rho((\sigma - 1)(1 - \eta) + \sigma\rho)}{((\sigma - 1)(1 - \eta) + \sigma\rho(1 - \omega_h - \omega_m))^2 \left(1 - \frac{(\sigma - 1)(1 - \eta)\delta\omega_h - \sigma\rho\delta\omega_m}{(\sigma - 1)(1 - \eta) + \sigma\rho(1 - \omega_h - \omega_m)}\right)^2}, \end{aligned}$$

as well as

$$\begin{aligned} \frac{1}{\sigma} \frac{\partial \ln(\Gamma_{OV}/\Gamma_{OO})}{\partial \omega_m \partial \delta} &= -\frac{(1 - \omega_h)((\sigma - 1)(1 - \eta) + \sigma\rho)}{((\sigma - 1)(1 - \eta) + \sigma\rho(1 - \omega_h - \omega_m))^2} \\ &\times \frac{((\sigma - 1)(1 - \eta)(1 - \delta\omega_h) + \sigma\delta\rho(1 - \omega_h))}{(1 - \delta) \left(1 - \frac{(\sigma - 1)(1 - \eta)\delta\omega_h - \sigma\rho\delta\omega_m}{(\sigma - 1)(1 - \eta) + \sigma\rho(1 - \omega_h - \omega_m)}\right)^2} < 0. \end{aligned}$$

Thus,  $\partial \ln(\Gamma_{OV}/\Gamma_{OO})/\partial \omega_m$  cannot be higher than when evaluated at  $\delta = 0$ , at which it is clear from the above expression that this derivative is zero. In sum, we have  $\partial \ln(\Gamma_{OV}/\Gamma_{OO})/\partial \omega_m < 0$  for all  $\delta > 0$ .

The fact that  $\Gamma_{OV}/\Gamma_{OV}$  in (A.18) is increasing in  $\omega_h$  and decreasing in  $\omega_m$  immediately implies that this same ratio is decreasing in headquarter contractibility  $\mu_h$  and increasing in manufacturing contractibility  $\mu_m$ , just as in the model with a single supplier. As mentioned in the main text, however, this does not imply that the ratio  $\Gamma_{OV}/\Gamma_{OO}$  is necessarily increasing in headquarter intensity  $\eta$ , since this parameter enters the formula (A.18) independently of how it shapes  $\omega_h$  and  $\omega_m$ .

Next, we show that the ratio  $\Gamma_{OV}/\Gamma_{OO}$  can only be lower than one if the degree of input substitutability as governed by  $\rho$  is above a *unique* certain threshold  $\hat{\rho} > 0$ . When that threshold is higher than one, then  $\Gamma_{OV}/\Gamma_{OO} > 1$  for all  $\rho \in (0, 1]$ . To show this, I begin by noting that when  $\rho \rightarrow 0$ , the ratio  $\Gamma_{OV}/\Gamma_{OO}$  in (A.18) clearly goes to  $+\infty$ , and thus the ratio is higher than one. When  $\rho \rightarrow +\infty$ , the ratio  $\Gamma_{OV}/\Gamma_{OO}$  goes to

$$\frac{\Gamma_{OV}}{\Gamma_{OO}} = \left(1 + \frac{\delta\omega_m}{1 - \omega_h - \omega_m}\right)^{\sigma(1 - \omega_h - \omega_m)} (1 - \delta)^{\sigma\omega_m} < 1,$$

where the inequality follows from the fact that the expression is decreasing in  $\delta$  and equals 1 at  $\delta = 0$ . Hence, we have that  $\Gamma_{OV}/\Gamma_{OO} > 1$  for sufficiently low  $\rho$ , and  $\Gamma_{OV}/\Gamma_{OO} < 1$  for a high enough  $\rho$ . To demonstrate the existence of a unique threshold  $\hat{\rho} > 0$  at which  $\Gamma_{OV}/\Gamma_{OO} = 1$ , we note that tedious differentiation delivers

$$\frac{1}{\sigma} \frac{\partial \ln(\Gamma_{OV}/\Gamma_{OO})}{\partial \rho} = \frac{\delta(\sigma-1)(1-\eta)}{((\sigma-1)(1-\eta) + \sigma\rho(1-\omega_h-\omega_m))} \times \left[ \frac{(1-\omega_h-\omega_m)\sigma(1-\omega_h)(\omega_h+\omega_m)}{(\sigma-1)(1-\eta) + \sigma\rho(1-\omega_h-\omega_m) - (\sigma-1)(1-\eta)\delta\omega_h + \sigma\rho\delta\omega_m} - \omega_h \frac{(\sigma-1)(1-\eta) + \sigma\rho(1-\omega_h-\omega_m)}{\sigma\rho^2 \left(1 + \frac{(\sigma-1)(1-\eta)\delta}{\sigma\rho}\right)} \right].$$

It can then be shown that the condition  $\partial \ln(\Gamma_{OV}/\Gamma_{OO})/\partial \rho = 0$  can be expressed as a quadratic equation

$$\rho^2 + b\rho + c = 0,$$

in which

$$c = -\frac{\omega_h(\sigma-1)^2(1-\eta)^2(1-\delta\omega_h)}{\sigma^2\omega_m(1-\omega_h-\omega_m)(1-\delta\omega_h)} < 0.$$

The fact that  $c$  is negative implies, however, that there can only be one positive solution ( $\rho > 0$ ) to this equation. Together with the limiting values  $\lim_{\rho \rightarrow 0}(\Gamma_{OV}/\Gamma_{OO}) = +\infty$  and  $\lim_{\rho \rightarrow +\infty}(\Gamma_{OV}/\Gamma_{OO}) < 1$ , we can thus conclude that  $\Gamma_{OV}/\Gamma_{OO} = 1$  for a unique value  $\hat{\rho} > 0$ .

It should be emphasized that this result does not imply that  $\Gamma_{OV}/\Gamma_{OO}$  is necessarily decreasing in  $\rho$  for all value of  $\rho \in (0, 1)$ . In fact, it is not difficult to construct examples in which  $\Gamma_{OV}/\Gamma_{OO}$  increases in  $\rho$  for a range of parameter values. For similar reasons, when studying the cross-partial derivative of  $\ln(\Gamma_{OV}/\Gamma_{OO})$  with respect to  $\rho$  and the levels of contractibility  $\mu_h$  and  $\mu_m$ , one can generate numerical examples in which these derivatives take positive or negative numbers.