A Theoretical Appendix

A.1 Optimal Sourcing Strategy in the Multi-Country Global Sourcing Model in Chapter 2

In this Appendix, I formally prove some statements related to the characterization of the optimal sourcing strategy in the multi-country global sourcing Model in Chapter 2.

Remember that the problem of choosing an optimal sourcing strategy is given by

$$\begin{aligned}\max_{\mathcal{J}_i(\varphi)} \pi_i\left(\varphi, \mathcal{J}_i\left(\varphi\right)\right) &= \left(a_{hi} w_i\right)^{-\eta(\sigma-1)} \left(\gamma \sum\nolimits_{k \in \mathcal{J}_i(\varphi)} T_k \left(\tau_{ik} w_k\right)^{-\theta}\right)^{(\sigma-1)(1-\eta)/\theta} B\varphi^{\sigma-1} \\ &\quad - w_i \sum\nolimits_{k \in \mathcal{J}_i(\varphi)} f_{ik}.\end{aligned}$$

With a discrete number of locations, we can rewrite the problem as follows:

$$\begin{aligned}\max_{I_1, I_2, ..., I_J \in \{0,1\}^J} \pi_i\left(\varphi, I_1, I_2, ..., I_J\right) &= \left(\sum_{k=1}^{J} I_k T_k \left(\tau_{ik} w_k\right)^{-\theta}\right)^{(\sigma-1)(1-\eta)/\theta} \widetilde{B}\varphi^{\sigma-1} \\ &\quad - w_i \sum_{k=1}^{J} I_k f_{ik}, \end{aligned} \tag{8.1}$$

where $\widetilde{B} = \left(a_{hi} w_i\right)^{-\eta(\sigma-1)} \gamma^{(\sigma-1)(1-\eta)} B$. The dummy variable I_j thus takes a value of 1 when $j \in \mathcal{J}_i\left(\varphi\right)$ and 0 otherwise.

The key thing to note is that, provided that $\left(\sigma - 1\right)\left(1 - \eta\right) > \theta$, the modified objective function in (8.1) features increasing differences in $\left(I_j, I_k\right)$ for $j, k \in \{1, ..., J\}$ such that $j \neq k$, and also features increasing differences in $\left(I_j, \varphi\right)$ for any $j \in \{1, ..., J\}$. Invoking standard results in monotone comparative statics, we can then conclude that for $\varphi_1 \geq \varphi_0$, we must have $\left(I_1^*\left(\varphi_1\right), I_2^*\left(\varphi_1\right), ..., I_J^*\left(\varphi_1\right)\right) \geq \left(I_1^*\left(\varphi_0\right), I_2^*\left(\varphi_0\right), ..., I_J^*\left(\varphi_0\right)\right)$. Naturally, this rules out a situation in which $I_j^*\left(\varphi_1\right) = 0$ but $I_j^*\left(\varphi_0\right) = 1$, and thus we can conclude that $\mathcal{J}_i\left(\varphi_0\right) \subseteq \mathcal{J}_i\left(\varphi_1\right)$ for $\varphi_1 \geq \varphi_0$.

A.2 Comparative Statics of the Global Sourcing Model in Chapter 4

In this Appendix, I will provide formal proofs for some comparative statics mentioned in Chapters 4 and 5. When a result has been proved in an existing paper, I will simply refer the reader to that paper.

Derivation of a General Formula for the Offshoring Share

In the first part of Chapter 5, I studied the determinants of the cross-section of offshoring shares. In Chapter 4 I derived a formula for this share but under the strong assumptions of complete contracting in the North, 'totally' incomplete contracting in the South, a single input, and symmetric bargaining. In Chapter 5, I appealed to a general formula that applies to all the extensions of the two-country model developed in Chapter 4. Let me now provide more details on that derivation.

As explained in Chapter 4, the share of foreign input purchases in total input purchases typically depends on how these inputs are priced in the presence of incomplete contracting and renegotiation. Below, I stick to the assumption in the main text that the ratio of input expenditures to sale revenue is common for firms sourcing domestically and offshoring. As a result, the offshoring share is identical to the fraction of industry sales captured by firms offshoring intermediate inputs. With a constant price elasticity of demand $\sigma > 1$, firm revenues are in turn a multiple σ of operating profits. Operating profits are in turn equal to overall profits plus fixed costs, or

$$\begin{aligned}\pi_D(\varphi) + f_D w_N &= (w_N)^{1-\sigma} B\Gamma_D \varphi^{\sigma-1} \\ \pi_O(\varphi) + f_O w_N &= \left((w_N)^{\eta} (\tau w_S)^{1-\eta}\right)^{1-\sigma} B\Gamma_O \varphi^{\sigma-1}.\end{aligned}$$

Assuming selection into offshoring – i.e., condition (2.21) in Chapter 2, we can define the thresholds $\tilde{\varphi}_O > \tilde{\varphi}_D$ satisfying $\pi_D(\tilde{\varphi}_D) = 0$ and $\pi_O(\tilde{\varphi}_O) = \pi_D(\tilde{\varphi}_O)$. It is straightforward to verify that

$$\frac{\tilde{\varphi}_O}{\tilde{\varphi}_D} = \left[\frac{f_O/f_D - 1}{\frac{\Gamma_O}{\Gamma_D}\left(\frac{w_N}{\tau w_S}\right)^{(1-\eta)(\sigma-1)} - 1}\right]^{1/(\sigma-1)}. \tag{8.2}$$

The share of revenues (and of input purchases) accounted for by offshoring firms is then given by

$$\Upsilon_O = \frac{\int_{\tilde{\varphi}_O}^{\infty} \left((w_N)^{\eta} (\tau w_S)^{1-\eta} \right)^{1-\sigma} B\Gamma_O \varphi^{\sigma-1} dG(\varphi)}{\int_{\tilde{\varphi}_D}^{\tilde{\varphi}_O} (w_N)^{1-\sigma} B\Gamma_D \varphi^{\sigma-1} dG(\varphi) + \int_{\tilde{\varphi}_O}^{\infty} \left((w_N)^{\eta} (\tau w_S)^{1-\eta} \right)^{1-\sigma} B\Gamma_O \varphi^{\sigma-1} dG(\varphi)}.$$

Assuming a Pareto distribution of productivity – i.e., $G(\varphi) = 1 - \left(\underline{\varphi}/\varphi\right)^{\kappa}$ for $\varphi \geq \underline{\varphi} > 0$ –, this expression further simplifies to

$$\Upsilon_O = \frac{\frac{\Gamma_O}{\Gamma_D} \left(\frac{w_N}{\tau w_S} \right)^{(1-\eta)(\sigma-1)}}{\left(\frac{\tilde{\varphi}_O}{\tilde{\varphi}_D} \right)^{\kappa-(\sigma-1)} - 1 + \frac{\Gamma_O}{\Gamma_D} \left(\frac{w_N}{\tau w_S} \right)^{(1-\eta)(\sigma-1)}},$$

where $\tilde{\varphi}_O/\tilde{\varphi}_D$ is given in 8.2. This corresponds to the general offshoring share equation (5.2) in Chapter 5. This formula is identical to the one applying in the complete-contracting case except for the term Γ_O/Γ_D.

With this expression in hand, we can next turn to the study of comparative statics in the different variants of the global sourcing model developed in Chapter 4. Below I will focus on how the different parameters of the model shape the ratio Γ_O/Γ_D, which differs across variants of the model. As argued in the main text, leaving aside this term Γ_O/Γ_D, the share Υ_O is increasing in w_N/w_S and σ, and decreasing in τ, f_O/f_D, κ and η (these results are straightforward to prove by simple differentiation (making use of $\kappa \geq \sigma - 1$).

Symmetric Nash Bargaining Model

Consider first the basic model with complete contracting in the North, 'totally' incomplete contracting in the South, a single input, and symmetric bargaining. This implies $\Gamma_D = 1$ and thus (see eq. (4.10)).

$$\frac{\Gamma_O}{\Gamma_D} = (\sigma + 1) \left(\frac{1}{2} \right)^{\sigma}.$$

But note that

$$\frac{\partial (\Gamma_O/\Gamma_D)}{\partial \sigma} = -\left(\frac{1}{2} \right)^{\sigma} ((1+\sigma)\ln 2 - 1) < 0,$$

and thus the offshoring share is lower in higher elasticity sectors on account of the effect of contractual frictions. This effect is of the opposite sign to the 'standard' one operating in the complete-contracting case, and thus the overall effect of σ on the offshoring share Υ_O is ambiguous.

Generalized Nash Bargaining Model

Let us now turn to the basic model with generalized Nash bargaining. Again we have $\Gamma_D = 1$ and thus

$$\frac{\Gamma_O}{\Gamma_D} = \Gamma_\beta \equiv (\sigma - (\sigma - 1)(\beta\eta + (1-\beta)(1-\eta)))\left(\beta^\eta (1-\beta)^{1-\eta}\right)^{\sigma-1},$$

as indicated by equation (4.14).

As mentioned in the main text, the effects of β and η on Γ_β are ambiguous and interact with each other. More specifically, we next show that Γ_β is decreasing in η when $\beta < 1/2$, while it is increasing in η when $\beta > 1/2$. To see this, first note that

$$\left.\frac{\partial \ln \Gamma_\beta}{\partial \eta}\right|_{\eta=0} = (\sigma - 1)\left(\frac{1-2\beta}{1-\beta+\sigma\beta} + \ln\left(\frac{\beta}{1-\beta}\right)\right)$$

and

$$\left.\frac{\partial \ln \Gamma_\beta}{\partial \eta}\right|_{\eta=1} = (\sigma - 1)\left(\frac{1-2\beta}{\sigma(1-\beta)+\beta} + \ln\left(\frac{\beta}{1-\beta}\right)\right).$$

It is not hard to show that each of these two expressions is negative for $\beta < 1/2$ and positive for $\beta > 1/2$. In particular, one can use $1 - x + \ln x \leq 0$ and $\ln 1/x - (1-x) \geq 0$, with $x = \beta/(1-\beta)$ to rewrite these expressions in a way that makes this obvious by inspection.[1] Next, notice that

$$\frac{\partial^2 \ln \Gamma_\beta}{\partial \eta^2} = -\frac{(\sigma-1)^2 (1-2\beta)^2}{(\sigma - (\sigma-1)(\beta\eta + (1-\beta)(1-\eta)))^2} < 0.$$

In sum, we have that when $\beta < 1/2$, $\partial \ln \Gamma_\beta / \partial \eta < 0$ when evaluated at $\eta = 0$, while for $\beta > 1/2$, $\partial \Gamma_\beta / \partial \eta > 0$ when evaluated at $\eta = 1$. Together with the concavity of Γ_β, we can then conclude that $(\partial \ln \Gamma_\beta / \partial \eta)(\beta - 1/2) \geq 0$ for all η, with strict inequality for $\beta \neq 1/2$. The practical relevance of this result is that it complicates the overall comparative static of the offshoring share Υ_O with respect to η (remember that under complete contracting, Υ_O is unambiguously decreasing in η).

We next consider how the ambiguous effect of changes in β interacts with η. I begin by noting that simple differentiation delivers

$$\frac{\partial \ln(\Gamma_O/\Gamma_D)}{\partial \beta} = (\sigma - 1)\frac{\eta(1-\eta) + \sigma\eta^2 - ((\sigma-1)\eta + 1)2\eta\beta + \sigma(2\eta - 1)\beta^2}{\beta(1-\beta)(\sigma - (\sigma-1)(\beta\eta + (1-\beta)(1-\eta)))}$$

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[1] For completenetss, note that $\frac{1-2\beta}{1-\beta+\sigma\beta} + \ln\left(\frac{\beta}{1-\beta}\right) = (2\beta - 1)\frac{\sigma\beta}{(1-\beta)(1-\beta+\sigma\beta)} + \left(1 - \frac{\beta}{1-\beta}\right) + \ln\left(\frac{\beta}{1-\beta}\right) = (2\beta-1)\frac{(\sigma-2)\beta+1}{\beta((\sigma-1)\beta+1)} + \ln\left(\frac{\beta}{1-\beta}\right) - \left(1 - \frac{1-\beta}{\beta}\right)$ and $\frac{1-2\beta}{\sigma(1-\beta)+\beta} + \ln\left(\frac{\beta}{1-\beta}\right) = (2\beta-1)\frac{\sigma-1-(\sigma-2)\beta}{(1-\beta)(\sigma+\beta-\sigma\beta)} + \left(1 - \frac{\beta}{1-\beta}\right) + \ln\left(\frac{\beta}{1-\beta}\right) = \sigma(2\beta-1)\frac{1-\beta}{\beta(\sigma+\beta-\sigma\beta)} + \ln\left(\frac{\beta}{1-\beta}\right) - \left(1 - \frac{1-\beta}{\beta}\right)$.

and

$$\frac{\partial^2 \ln\left(\Gamma_O/\Gamma_D\right)}{\partial\beta^2} = -\left(\sigma-1\right)^2\left(\frac{\left(2\eta-1\right)^2}{\left(\sigma-\left(\sigma-1\right)\left(\beta\eta+\left(1-\beta\right)\left(1-\eta\right)\right)\right)^2}+\frac{\eta\left(1-\beta\right)+\beta\left(\beta-\eta\right)}{\left(\sigma-1\right)\beta^2\left(1-\beta\right)^2}\right)<0.$$

Thus, Γ_O/Γ_D is maximized for the value(s) of β that solve the quadratic equation in the numerator of $\partial \ln\left(\Gamma_O/\Gamma_D\right)/\partial\beta$. It turns out that there is only one solution β^* of this quadratic equation satisfying $\beta^* \in [0,1]$. Rearranging this solution, we find equation (4.15) in 4, which makes it clear that β^* is increasing in η.

Consider finally how the elasticity of demand affects the ratio Γ_O/Γ_D. Simple (though tedious) differentiation confirms first that

$$\frac{\partial^2\left(\ln\left(\Gamma_O/\Gamma_D\right)\right)}{\partial\sigma^2} = -\frac{\left(\beta+\eta-2\beta\eta\right)^2}{\left(\sigma-\left(\sigma-1\right)\left(\beta\eta+\left(1-\beta\right)\left(1-\eta\right)\right)\right)^2}<0.$$

Hence $\partial \ln\left(\Gamma_O/\Gamma_D\right)/\partial\sigma$ is bounded above by the value of this derivative when evaluated at the lowest possible value of σ, namely $\sigma = 1$. But note that

$$\left.\frac{\partial \ln\left(\Gamma_O/\Gamma_D\right)}{\partial\sigma}\right|_{\sigma=1} = \beta+\eta-2\beta\eta+\ln\left(\beta^{\eta}\left(1-\beta\right)^{1-\eta}\right).$$

To evaluate this expression, notice that it increases in η when $\beta > 1/2$, while it decreases in η when $\beta < 1/2$.[2] Furthermore, the expression equals $\beta+\ln\left(1-\beta\right) \leq 0$ when $\eta = 0$, $1-\beta+\ln\left(\beta\right) \leq 0$ when $\eta = 1$, and $\frac{1}{2}+\ln\left(\frac{1}{2}\right) < 0$ when $\beta = 1/2$. We can thus conclude that $\partial \ln\left(\Gamma_O/\Gamma_D\right)/\partial\sigma < 0$ for $\sigma > 1$.

Limitations on Ex-Ante Transfers: Financial Constraints

Remember that the case in which M cannot transfer to F ex-ante more than a share ϕ of his or her ex-post rents, delivered

$$\frac{\Gamma_O}{\Gamma_D} = \Gamma_\phi \equiv \left(\sigma+\phi-\left(\sigma-1\right)\left(1-\phi\right)\eta\right)\left(\frac{1}{2}\right)^{\sigma},$$

since again we assumed $\Gamma_D = 1$. It is obvious from this expression that Γ_ϕ increases in ϕ and η, and these effects interact in a positive matter, or $\partial^2\Gamma_\phi/\left(\partial\phi\partial\eta\right) > 0$. The positive effect of η on Γ_ϕ again renders ambiguous the overall effect of headquarter intensity on the offshoring share Υ_O (with complete contracting, Υ_O is unambiguously decreasing in η).

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[2] This in turn can be shown again by applying the inequalities $1-x+\ln x \leq 0$ and $\ln 1/x-\left(1-x\right) \geq 0$ with $x = \beta/\left(1-\beta\right)$, and decomposing $1-2\beta+\ln\frac{\beta}{1-\beta} = \beta\frac{2\beta-1}{1-\beta}+1-\frac{\beta}{1-\beta}+\ln\frac{\beta}{1-\beta} = \left(2\beta-1\right)\frac{1-\beta}{\beta}-\left(1-\frac{1-\beta}{\beta}\right)+\ln\frac{\beta}{1-\beta}$.

Consider next the effect of the demand elasticity σ. Straightforward differentiation delivers

$$\frac{\partial \ln \Gamma_\phi}{\partial \sigma} = \frac{(1-\eta+\phi\eta)}{(\sigma+\phi-(\sigma-1)(1-\phi)\eta)} - \ln 2,$$

as well as $\partial^2 \ln \Gamma_\phi / \partial \sigma^2 < 0$. It is then straightforward to show that for a sufficiently high σ, we necessarily have $\partial \ln \Gamma_\phi / \partial \sigma < 0$. In fact, the weak condition $\sigma + \phi > (\ln 2)^{-1} = 1.4427$ is sufficient for this inequality to hold, regardless of the value of η.

Partial Contractibility

In the extension of the model with partial contractibility in both countries, I alluded to the results in Antràs and Helpman (2008) to motivate the following expressions for the index of contracting distortions under domestic sourcing and offshoring:

$$\Gamma_D = \left(\frac{\sigma}{\sigma-(\sigma-1)(1-\mu_N)}+1\right)^{\sigma-(\sigma-1)(1-\mu_N)}\left(\frac{1}{2}\right)^{\sigma}; \tag{8.3}$$

$$\Gamma_O = \left(\frac{\sigma}{\sigma-(\sigma-1)(1-\mu_S)}+1\right)^{\sigma-(\sigma-1)(1-\mu_S)}\left(\frac{1}{2}\right)^{\sigma}, \tag{8.4}$$

where

$$\mu_N \equiv \eta\mu_{hN} + (1-\eta)\mu_{mN};$$
$$\mu_S \equiv \eta\mu_{hS} + (1-\eta)\mu_{mS}.$$

In fact, these expressions are a special case of those that apply in the framework in Antràs and Helpman (2008). Because I will be referring to these more general results repeatedly in the derivations below, it might be useful to sketch here the steps that lead to that more general formula.

With that in mind, consider the following generalization of the problem in (4.19) after substitution of the participation constraint pinning down the ex-ante transfer:

$$\max_{h_c,h_n,m_c,m_n,s} \quad R - w_N\left(\mu_{hj}h_c + \left(1-\mu_{hj}\right)h_n + \mu_{mj}m_c + \left(1-\mu_{mj}\right)m_n\right) - s$$
$$\text{s.t.} \quad h_n = \arg\max_h\{\beta_h R - w_N\left(1-\mu_{hj}\right)h_n\}$$
$$m_n = \arg\max_m\{\beta_m R - c_j\left(1-\mu_{mj}\right)m_n\},$$

where revenue is given by

$$R = B^{1/\sigma}\sigma(\sigma-1)^{-(\sigma-1)/\sigma}\varphi^{(\sigma-1)/\sigma} \times \left(\frac{(h_c)^{\mu_{hj}}(h_n)^{1-\mu_{hj}}}{\eta}\right)^{(\sigma-1)\eta/\sigma}\left(\frac{(m_c)^{\mu_{mj}}(m_n)^{1-\mu_{mj}}}{1-\eta}\right)^{(\sigma-1)(1-\eta)/\sigma} \quad (8.5)$$

and where $c_j = w_N$ when $j = N$ and $c_j = \tau w_S$ when $j = S$. The problem above thus cover the cases of symmetric and generalized Nash bargaining, but it also encompasses environments with partial relationship-specificity in which F and M only bargain over a fraction of revenue ex-post, and thus $\beta_h + \beta_m < 1$. And, as discussed below, this formulation will also prove useful in the characterization of the equilibrium under multiple suppliers.

In order to derive the formula for profits associated with this more general problem, notice first that from the two constraints of the problem, we have

$$h_n = \frac{\beta_h(\sigma-1)\eta}{\sigma w_N}R$$
$$m_n = \frac{\beta_m(\sigma-1)\eta}{\sigma w_j}R.$$

Plugging these expressions into (8.5), delivers

$$R = \left(B^{1/\sigma}\sigma(\sigma-1)^{-(\sigma-1)/\sigma}\varphi^{(\sigma-1)/\sigma}\right)^{\frac{\sigma}{\sigma-(\sigma-1)\left(1-\mu_j\right)}}\left(\frac{h_c}{\eta}\right)^{\frac{(\sigma-1)\eta\mu_{hj}}{\sigma-(\sigma-1)\left(1-\mu_j\right)}}\left(\frac{m_c}{1-\eta}\right)^{\frac{(\sigma-1)(1-\eta)\mu_{mj}}{\sigma-(\sigma-1)\left(1-\mu_j\right)}} \times \left(\frac{\beta_h(\sigma-1)}{\sigma w_N}\right)^{\frac{(\sigma-1)\eta\left(1-\mu_{hj}\right)}{\sigma-(\sigma-1)\left(1-\mu_j\right)}}\left(\frac{\beta_m(\sigma-1)}{\sigma w_j}\right)^{\frac{(\sigma-1)(1-\eta)\left(1-\mu_{mj}\right)}{\sigma-(\sigma-1)\left(1-\mu_j\right)}}. \quad (8.6)$$

Given the Cobb-Douglas structure, we can then characterize the choice of contractible investments as satisfying

$$h_c = \frac{(\sigma-1)\eta\left(1-\frac{(\sigma-1)}{\sigma}\left(\beta_h\eta\left(1-\mu_{hj}\right)+\beta_m(1-\eta)\left(1-\mu_{mj}\right)\right)\right)}{\left(\sigma-(\sigma-1)\left(1-\mu_j\right)\right)w_N}R \quad (8.7)$$

$$m_m = \frac{(\sigma-1)(1-\eta)\left(1-\frac{(\sigma-1)}{\sigma}\left(\beta_h\eta\left(1-\mu_{hj}\right)+\beta_m(1-\eta)\left(1-\mu_{mj}\right)\right)\right)}{\left(\sigma-(\sigma-1)\left(1-\mu_j\right)\right)w_j}R \quad (8.8)$$

As a result, operating profits are given by

$$\left(\frac{\sigma-(\sigma-1)\left(\beta_h\left(1-\mu_{hj}\right)+\beta_m\left(1-\mu_{mj}\right)\right)}{\sigma-(\sigma-1)\left(1-\mu_j\right)}\right)\frac{R}{\sigma},$$

where R can be solved by plugging the above expression (8.7) and (8.8) into (8.6). This delivers, after some manipulations

$$\begin{aligned} R &= \sigma B \left((w_N)^{\eta} (w_j)^{1-\eta} \right)^{1-\sigma} \varphi^{\sigma-1} (\beta_h)^{(\sigma-1)\eta\left(1-\mu_{hj}\right)} (\beta_m)^{(\sigma-1)(1-\eta)\left(1-\mu_{mj}\right)} \\ & \quad \times \left(\frac{\sigma - (\sigma-1)\left(\beta_h \eta \left(1-\mu_{hj}\right) + \beta_m (1-\eta)\left(1-\mu_{mj}\right)\right)}{\sigma - (\sigma-1)\left(1-\mu_j\right)} \right)^{(\sigma-1)\mu_j}, \end{aligned}$$

and thus

$$\begin{aligned} \pi_D(\varphi) + f_D w_N &= (w_N)^{1-\sigma} B \Gamma_D \varphi^{\sigma-1} \\ \pi_O(\varphi) + f_O w_N &= \left((w_N)^{\eta} (\tau w_S)^{1-\eta} \right)^{1-\sigma} B \Gamma_O \varphi^{\sigma-1}, \end{aligned}$$

where

$$\begin{aligned} \Gamma_\ell &= \left(\frac{\sigma - (\sigma-1)\left(\beta_h \eta \left(1-\mu_{hj}\right) + \beta_m (1-\eta)\left(1-\mu_{mj}\right)\right)}{\sigma - (\sigma-1)\left(1-\mu_j\right)} \right)^{\sigma-(\sigma-1)\left(1-\mu_j\right)} \\ & \quad \times (\beta_h)^{(\sigma-1)\eta\left(1-\mu_{hj}\right)} (\beta_m)^{(\sigma-1)(1-\eta)\left(1-\mu_{mj}\right)} \end{aligned} \tag{8.9}$$

captures the contractual frictions associated with the sourcing options $\ell = D$ and $\ell = O$, which entail manufacturing in country $j = N$ and country $j = S$, respectively. Setting $\beta_h = \beta_m = 1/2$, it is straightforward to verify that equation (8.9) reduces to equations (8.3) and (8.4) above.

Having derived these equations, we can next turn to discussing some key comparative statics. Below, I will focus on an analysis of the general formula (8.9), with the understanding that the results obtained below also apply to the particular case with $\beta_h = \beta_m = 1/2$. Consider first the effect of the indices of contractibility μ_{hj} and μ_{mj}, and its weighted average μ_j. As shown in Antràs and Helpman (2008) (see the proof of their Proposition 1), Γ_ℓ is necessarily non-decreasing in each of these parameters. The proof in that paper is rather cumbersome, so it may be worth offering a much simpler proof here. Consider the case of an increase in μ_{hj} (the derivations associated with a change in μ_{mj} are analogous). Taking logs of (8.9), differentiating and rearranging terms, we can write

$$\frac{\partial \ln \Gamma_\ell}{\partial \mu_{hj}} = \eta (\sigma - 1) \left(-\ln \mathcal{Q} - (1 - \mathcal{Q}) - \ln \beta_h - (1 - \beta_h) \right) + \mathcal{W}, \tag{8.10}$$

where

$$\mathcal{Q} = \frac{\sigma - (\sigma-1)\left(1-\mu_j\right)}{\sigma - (\sigma-1)\left(\beta_h \eta \left(1-\mu_{hj}\right) + \beta_m (1-\eta)\left(1-\mu_{mj}\right)\right)}$$

and

$$\mathcal{W} = \eta (\sigma - 1)^2 (1 - \beta_h) \frac{(1-\eta)\left(1-\mu_{mj}\right)(1-\beta_m) + \eta\left(1-\mu_{hj}\right)(1-\beta_h)}{\sigma - (\sigma-1)\left(\beta_h \eta \left(1-\mu_{hj}\right) + \beta_m (1-\eta)\left(1-\mu_{mj}\right)\right)}.$$

It is clear that the second term is positive, while the first one is non-negative too because $-\ln x - (1-x) \geq 0$ for all x. Thus, $\partial \ln \Gamma_\ell / \partial \mu_{hj} \geq 0$.

It is also clear from inspection of equation (8.9), that as stated in the main text, the effect of improvements in contractibility interacts with the headquarter intensity of production depending on the source of these changes in contractibility. Increases in μ_{hj} will be particularly beneficial when η is high, while the converse is true for μ_{mj}. For the same reason, and as in the model with totally incomplete contracting and generalized Nash bargaining, the effect of changes in headquarter intensity on Γ_ℓ is ambiguous.

Let us now turn to the effect of the elasticity of demand σ on Γ_ℓ. Tedious differentiation of Γ_ℓ delivers

$$\frac{\partial^2 \ln \Gamma_\ell}{\partial \sigma^2} = -\frac{(1-\mu-\eta(1-\mu_h)\beta_h - (1-\eta)(1-\mu_m)\beta_m)^2}{(1-\mu+\sigma\mu)(\sigma-(\sigma-1)(\beta_h\eta(1-\mu_h)+\beta_m(1-\eta)(1-\mu_m)))^2} < 0$$

and

$$\left.\frac{\partial \ln \Gamma}{\partial \sigma}\right|_{\sigma=1} = 1 - \mu + \eta(1-\mu_h)(\ln \beta_h - \beta_h) + (1-\eta)(1-\mu_m)(\ln \beta_m - \beta_m) \leq 0.$$

To prove the negative sign in the second equation, note that this expression is maximized when $\beta_h = \beta_m = 1$, and at that level $\partial \ln \Gamma / \partial \sigma|_{\sigma=1} = 1 - \mu - \eta(1-\mu_h) - (1-\eta)(1-\mu_m) = 0$. In light of these results, we can conclude that $\partial \ln \Gamma / \partial \sigma < 0$ for all $\sigma > 1$, and thus contractual frictions are again aggravated by high demand elasticities in this variant of the model.

We next show how the effects of contractibility and the elasticity of demand interact with each other. In particular, differentiating $\partial \ln \Gamma_\ell / \partial \mu_{hj}$ in (8.10) with respect to σ, we find:

$$\frac{\partial^2 \ln \Gamma_\ell}{\partial \mu_{hj} \partial \sigma} = \frac{1}{(\sigma-1)} \frac{\partial \ln \Gamma_\ell}{\partial \mu_{hj}} + \eta(\sigma-1)\frac{\partial(-\ln \mathcal{Q} - (1-\mathcal{Q}))}{\partial \sigma} + \frac{\partial \mathcal{W}}{\partial \sigma}.$$

We have established before that the first term is non-negative. Differentiating the second and third terms, we find

$$\frac{\partial\left(\ln \mathcal{Q}^{-1} - 1 + \mathcal{Q}\right)}{\partial \sigma} = \frac{(\sigma-1)\left(\frac{(1-\eta)(1-\mu_m)(1-\beta_m)+\eta(1-\mu_h)(1-\beta_h)}{\sigma-(\sigma-1)(\beta_h\eta(1-\mu_h)+\beta_m(1-\eta)(1-\mu_m))}\right)^2}{(\sigma-(\sigma-1)(1-\eta\mu_h-(1-\eta)\mu_m))}$$

and

$$\frac{\partial \mathcal{W}}{\partial \sigma} = (1 - \beta_h) \frac{(1-\eta)(1-\mu_m)(1-\beta_m) + \eta(1-\mu_h)(1-\beta_h)}{(\sigma - (\sigma-1)(\beta_h \eta (1-\mu_h) + \beta_m (1-\eta)(1-\mu_m)))^2},$$

and thus these terms are non-negative as well. In sum, we can conclude that $\frac{\partial^2 \ln \Gamma_\ell}{\partial \mu_{hj} \partial \sigma} \geq 0$, as stated in the main text. Notice, that the result is *not* particular to the special case $\beta_h = \beta_m = 1/2$, nor does it require $\beta_h + \beta_m = 1$. It is worth pointing out that it is important that we are considering the partial derivative of the logarithm of Γ_ℓ. Computing $\frac{\partial^2 \Gamma_\ell}{\partial \mu_{hj} \partial \sigma}$, we find that this expression may take negative values for some parameter values. This justifies the use of logarithms of import flows in certain empirical specifications in Chapter 5, as discussed in the main text.

I have thus far focused on providing formal proofs of the results mentioned in Chapter 4, which are key for interpreting the cross-country, cross-industry results in the second part of Chapter 5. The first part of that chapter focuses on studying the determinants of the offshoring share Υ_O, which in turn depend on the ratio Γ_O/Γ_D:

$$\frac{\Gamma_O}{\Gamma_D} = \frac{\left(\frac{\sigma-(\sigma-1)(\beta_h\eta(1-\mu_{hS})+\beta_m(1-\eta)(1-\mu_{mS}))}{\sigma-(\sigma-1)(1-\mu_S)}\right)^{\sigma-(\sigma-1)(1-\mu_S)}}{\left(\frac{\sigma-(\sigma-1)(\beta_h\eta(1-\mu_{hN})+\beta_m(1-\eta)(1-\mu_{mN}))}{\sigma-(\sigma-1)(1-\mu_N)}\right)^{\sigma-(\sigma-1)(1-\mu_N)}} \times (\beta_h)^{(\sigma-1)\eta(\mu_{hN}-\mu_{hS})} (\beta_m)^{(\sigma-1)(1-\eta)(\mu_{mN}-\mu_{mS})}.$$

From the results above, it is immediate that Γ_O/Γ_D is increasing in μ_S and its components μ_{hS} and μ_{mS}, and decreasing in μ_N and its components μ_{hN} and μ_{mN}. Less trivially, we can also use the results above to show that Γ_O/Γ_D is decreasing in the elasticity of demand σ provided that contract enforcement is higher in domestic transactions vis a vis offshoring transactions. In particular, notice that

$$\frac{\partial \ln(\Gamma_O/\Gamma_D)}{\partial \sigma} = \frac{\partial \ln(\Gamma_O)}{\partial \sigma} - \frac{\partial \ln(\Gamma_D)}{\partial \sigma},$$

and provided that $\mu_{hN} \geq \mu_{hS}$ and $\mu_{mN} \geq \mu_{mS}$, we can appeal to the above results $\frac{\partial^2 \ln \Gamma_\ell}{\partial \mu_{hj} \partial \sigma} \geq 0$ and $\frac{\partial^2 \ln \Gamma_\ell}{\partial \mu_{mj} \partial \sigma} \geq 0$ to conclude that $\partial \ln(\Gamma_O/\Gamma_D)/\partial \sigma \leq 0$.

Finally, it is important to emphasize that our results above do not suggest that offshoring shares will be higher for more "contractible" goods. To see this, suppose that contractibility in the South is always a fraction $\delta < 1$ of the one in the North, so we can write $\mu_{hS}/\mu_{hN} = \mu_{mS}/\mu_{mN} = \delta$. For the special case, $\beta_h = \beta_m = 1/2$, we then have

$$\frac{\Gamma_O}{\Gamma_D} = \frac{\left(\frac{\sigma}{\sigma-(\sigma-1)(1-\delta\mu_N)}+1\right)^{\sigma-(\sigma-1)(1-\delta\mu_N)}}{\left(\frac{\sigma}{\sigma-(\sigma-1)(1-\mu_N)}+1\right)^{\sigma-(\sigma-1)(1-\mu_N)}}.$$

Increases in μ_N can then be interpreted as overall increases in the contractibility of goods, since they affect their contractibility proportionately, regardless of the country where production takes place. It is not hard to confirm, however, that the effect of μ_N on the expression above is non-monotonic. For instance, if one sets $\sigma = 10$ and $\delta = 0.9$, Γ_O/Γ_D is lower when $\mu_N = 0.7$ than when either $\mu_N = 0.5$ or $\mu_N = 0.9$.

Relationship-Specificity

As discussed in the main text, this is a special case of the more general Antràs-Helpman (2008) framework with $\beta_h = \beta_m = 1 - \epsilon/2$, with $\epsilon \in [0, 1]$. The results derived above for the case of partial contractibility thus continue to apply. Improvements in contractibility are associated with larger values of Γ_ℓ, the elasticity of demand σ affects Γ_ℓ negatively, and the positive "interaction" effect $\partial\left(\partial \ln \Gamma_\ell / \partial \mu_j\right) / \partial \sigma > 0$ continue to apply. Similarly, we have that the offshoring share is negatively impacted by the elasticity of demand σ on account of the term Γ_O/Γ_D (remember though that there is a positive counterbalancing effect that applies even in the complete-contracting case).

Let us then focus on the new comparative statics that emerge when introducing relationship-specificity. Consider first the direct effect of the specificity parameter ϵ. Simple differentiation of (4.24) delivers

$$\frac{\partial \ln \Gamma_\ell\left(\mu_j, \epsilon\right)}{\partial \epsilon} = -\frac{\sigma\epsilon\left(\sigma - 1\right)\left(1 - \mu_j\right)}{\left(2 - \epsilon\right)\left(2\left(1 - \mu_j\right) + \left(2 - \epsilon\right)\sigma\mu_j + \left(\sigma - 1 + \mu_j\right)\epsilon\right)} < 0$$

and

$$\frac{\partial^2 \ln \Gamma_\ell\left(\mu_j, \epsilon\right)}{\partial \epsilon \partial \mu_j} = \frac{2\sigma^2\epsilon\left(\sigma - 1\right)}{\left(2 - \epsilon\right)\left(2\left(1 - \mu_j\right) + \left(2 - \epsilon\right)\sigma\mu_j + \left(\sigma - 1 + \mu_j\right)\epsilon\right)^2} > 0,$$

as stated in the main text. Hence, profitability is decreasing in specificity, and improvements in contractibility are particularly profitability-enhancing at high levels of specificity. Furthermore, we can use the latter result to conclude that

$$\frac{\partial \ln\left(\Gamma_O/\Gamma_D\right)}{\partial \epsilon} = \frac{\partial \ln\left(\Gamma_O\right)}{\partial \epsilon} - \frac{\partial \ln\left(\Gamma_D\right)}{\partial \epsilon} \leq 0$$

$\mu_{hN} \geq \mu_{hS}$ and $\mu_{mN} \geq \mu_{mS}$. In words, whenever contract enforcement is higher in domestic transactions relative to offshore transactions, higher levels of specificity tend to be associated with lower offshoring shares Υ_O.

Multiple Inputs and Multilateral Contracting

As mentioned in the main text, the equilibrium expressions of this variant of the model are analogous to those in Antràs and Helpman (2008) whenever $\beta_h = \beta_m = \sigma\rho / \left((\sigma - 1)(1 - \eta) + \sigma\rho \right)$. Plugging these values into (8.9) delivers equation (4.27). Because (4.27) is a special case of (8.9), we can conclude once again that $\partial\Gamma_\ell\left(\mu_j, \rho\right)/\partial\mu_j \geq 0$. Furthermore, we can also appeal to previous results to establish that $\partial\Gamma_\ell\left(\mu_j, \rho\right)/\partial\sigma < 0$. This latter comparative static result would appear to be complicated by the fact that β_h and β_m are now a function of σ. But since $\Gamma_\ell\left(\mu_j, \rho\right)$ in (8.9) is increasing in β_h and β_m, and each of these two shares is decreasing in σ, we can again conclude that $\partial\Gamma_\ell\left(\mu_j, \rho, \beta_h(\sigma), \beta_m(\sigma)\right)/\partial\sigma < 0$. In addition, the cross-partial derivative $\partial\left(\partial \ln \Gamma_\ell\left(\mu_j, \rho\right)/\partial\mu_j\right)/\partial\sigma$ continues to be positive, despite the dependence of β_h and β_m on σ. To see this, we can just appeal to equation (8.10) and note that each of the terms in that expression is decreasing in β_h, which in turn decreases in σ. More precisely, we have that (i) $-\ln\mathcal{Q} - (1 - \mathcal{Q})$ is decreasing in $\mathcal{Q}$ whenever $\mathcal{Q} < 1$, (ii) $\mathcal{Q}$ is indeed lower than 1 and is increasing in β_h, (iii) $-\ln\beta_h - (1 - \beta_h)$ is decreasing in β_h for $\beta_h < 1$, and (iv) $\mathcal{W}$ is decreasing in β_h.

We can next turn to the effects of ρ which is the new parameter introduced in this variant of the model. Simple differentiation of equation (4.27) indicates

$$\frac{\partial \ln \Gamma_\ell\left(\mu_j, \rho\right)}{\partial\rho} = \frac{(\sigma-1)^3 (1-\eta)^2 \left(1-\mu_j\right)}{\rho\left(\rho\sigma + (\sigma-1)(1-\eta)\right)\left(\left(\sigma - (\sigma-1)\left(1-\mu_j\right)\right)\rho + (\sigma-1)(1-\eta)\right)} > 0$$

and

$$\frac{\partial^2 \ln \Gamma_\ell\left(\mu_j, \rho\right)}{\partial\rho\partial\mu_j} = -(\sigma-1)^3 \frac{(1-\eta)^2}{\rho\left(\left(\sigma - (\sigma-1)\left(1-\mu_j\right)\right)\rho + (\sigma-1)(1-\eta)\right)^2} < 0,$$

which are the two key novel comparative statics highlighted in the main text of that section. Again, this last cross-partial derivative is useful in deriving predictions for the offshoring share Υ_O since for $\mu_{hN} \geq \mu_{hS}$ and $\mu_{mN} \geq \mu_{mS}$, this result implies $\frac{\partial \ln(\Gamma_O/\Gamma_D)}{\partial\rho} = \frac{\partial \ln(\Gamma_O)}{\partial\rho} - \frac{\partial \ln(\Gamma_D)}{\partial\rho} \geq 0$. In sum, whenever contract enforcement is higher in domestic transactions relative to offshore transactions, higher degrees of input substitutability tend to be associated with higher offshoring shares Υ_O.